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Fixed Probabilistic Evidence for Bayesian User Modeling

Rebai Rim[1,*], Maalej Mohamed Amin[1,2], Adel Mahfoudhi[1,2]

[1] University of Sfax,
ENIS, Computer Embedded System,
Tunisia

[2] University of Sfax, FSS,
Tunisia

rebai_rima@yahoo.fr

Abstract. Bayesian methodologies have emerged as a cornerstone of contemporary scientific inquiry, particularly within computer science, where their interpretability, adaptability, and robustness in managing uncertainty are highly valued. Continuous advancements in computation and methodology further propel their expanding influence. This is exemplified in Bayesian user modeling, where a system's adaptability is defined by its capacity to probabilistically deduce user requirements and dynamically optimize the presentation of information and user interaction pathways. Bayesian inference offers a structured framework for synthesizing observational data with prior knowledge, thereby facilitating robust analysis and decision-making in uncertain environments. This paper advocates for the enhancement of Bayesian user models through the integration of fixed probabilistic evidence, thereby extending the capabilities of conventional Bayesian networks. Our empirical results indicate that this augmented model functions as a powerful mechanism for discerning user preferences. Through a multi-metric evaluation of interface performance and a comparative study, we demonstrate that, in contrast to standard networks, Bayesian networks incorporating fixed probabilistic evidence unequivocally validate the essential role of evidence propagation, underscoring its critical importance for adaptive system design.

Keywords. Fixed probabilistic evidence, evaluation metrics, inference.

1 Introduction

The evolution of inference algorithms for Bayesian Networks has enabled their successful deployment across diverse domains including image recognition, medical diagnosis, natural language understanding, data mining, genetics, and search algorithms [9]. Central to these applications is Bayesian inference, which provides a mechanism for updating posterior probabilities.

The foundation of these inference methods lies in evidence—new information integrated into the network, often termed as observations or findings. The literature presents varied terminology for this concept, with descriptors such as 'soft evidence,' 'uncertain evidence,' 'probabilistic evidence,' and 'likelihood evidence' being employed [37]. A survey of recent literature reveals inconsistent usage of the term "soft evidence" over the past decade. While several studies equate it with likelihood evidence [37, 4, 8, 9, 24, 29], others have abandoned the term entirely [10].

This review, along with observations from Bayesian network software implementations, indicates that "soft evidence" remains an ambiguous concept due to its inconsistent application. Notably, researchers including Valtorta and colleagues [23, 39, 31, 32, 41, 42, 49], along with [28, 47], specifically use "soft evidence" to refer to fixed probabilistic evidence.

Bayesian user models provide a robust foundation for developing enhanced services in web applications, facilitating efficient user navigation and seamless information access. While various evaluation methodologies exist for assessing web applications [5,12,20,24], our

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study employs multiple performance metrics to evaluate an adaptive interface incorporating fixed probabilistic evidence.

This paper is structured as follows: Section 2 introduces soft evidence for Bayesian inference. In Section 3, we propose a refinement to the Bayesian model through the integration of fixed probabilistic evidence. Section 4 details the evaluation metrics for an adaptive user interface based on fixed probabilistic evidence.

2 Soft Evidence for Bayesian Inference

The effectiveness of Bayesian Networks in managing uncertainty has been particularly demonstrated in user modeling applications [49]. Specifically, Bayesian user modeling has emerged as a robust methodology for inferring user objectives and preferences [7, 11, 13].

These probabilistic models have been successfully implemented across various adaptive systems to anticipate user needs and goals [18, 48, 45]. Horvitz et al. [18] pioneered the application of Bayesian networks to deduce user objectives through analysis of interaction patterns and conditional probability models. In their foundational work, psychological observations of users in diverse contexts informed the initial network structure. In subsequent research [19], the authors developed an advanced system capable of predicting user intentions within inherently uncertain environments, further establishing the practical utility of Bayesian approaches for user modeling.

While probabilistic evidence shares with hard evidence the characteristic that its conveyed observation remains invariant post-propagation, it differs fundamentally from likelihood evidence in two primary aspects. First, regarding specification: probabilistic evidence is defined by a comprehensive probability distribution that integrates all available information ('all things considered'), whereas likelihood evidence is characterized by likelihood ratios established independently of any prior distribution ('without a prior') [2].

Soft evidence—alternatively termed fixed probabilistic evidence [2]—substantially enhances the computational power of Bayesian networks and expands their application domains [2, 9]. This form of evidence represents a natural extension of conventional hard evidence, capturing information originating from external sources beyond the process explicitly modeled within the Bayesian network framework.

The distinction between fixed and non-fixed probabilistic evidence becomes particularly evident following the integration and propagation of multiple evidentiary inputs. Non-fixed probabilistic evidence occurs when a variable's probability distribution is updated through the Bayesian network's application to a specific subpopulation of the original model domain. Conversely, fixed probabilistic evidence maintains its prescribed distribution unchanged—even when new evidence is introduced to other network nodes.

Bayesian network inference involves updating variable probability distributions in response to available evidence and observed findings. Evidence is typically represented as a set of findings categorized into two primary types: hard and soft. A hard finding asserts that a variable X assumes a specific value, with the complete set of such observations denoted by f [2]. In contrast, a soft finding is defined as a probability distribution over X's possible values, represented as R(X) [2, 9]. This probabilistic input type, termed soft evidence by Valtorta [49], has been extensively examined in subsequent research [2, 41, 42, 49, 50].

To elucidate the practical implications of these evidence types, we implement both forms within our Bayesian network model and examine their respective propagation dynamics through the network structure.

3 Fixed Probabilistic Evidence

3.1 Bayesian Networks

Bayesian networks (BNs) constitute a prominent class of probabilistic graphical models that represent dependencies among random variables using a directed acyclic graph (DAG) [7, 37, 49]. Structurally, a BN is composed of nodes, which correspond to random variables, directed edges that indicate conditional dependencies between these variables, and conditional probability distributions that quantify these relationships.

Formally, a Bayesian network is defined as a DAG comprising n nodes, denoted, $X = X_1, X_2, .., X_n$, where each node X_i is a random variable and the edges specify parent-child dependencies. The joint probability distribution over all variables factorizes according to the chain rule as follows:

$$P(X_1, X_2, .., X_n) = \prod_{i=1}^{n} P(X_i | pa(X_i)), \quad (1)$$

where $pa(X_i)$ denotes the set of parent nodes of X_i as defined by the incoming directed edges in the DAG.

Within the Bayesian framework, inference refers to the process of computing updated probability distributions for variables in the network conditioned on observed evidence.

The concept of "soft evidence" was originally introduced by Valtorta [49] and is formally defined as a probability distribution R(X) over a variable X. This form of evidence enables the representation of probabilistic knowledge about a variable's state when direct observation of its exact value is unavailable.

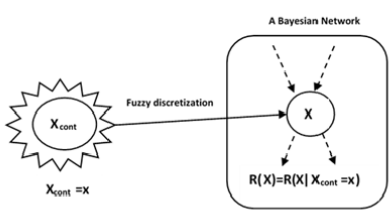

Fig. 1. The soft finding on X is generated by applying a fuzzy discretization process to the continuous variable X

3.2 A Framework for Integrating Fixed Probabilistic Evidence in Bayesian Models

The Junction Tree Algorithm serves as a general-purpose inference method applicable to diverse network topologies, including graphs with complex cyclic structures. This algorithm operates through a two-phase process: first, a

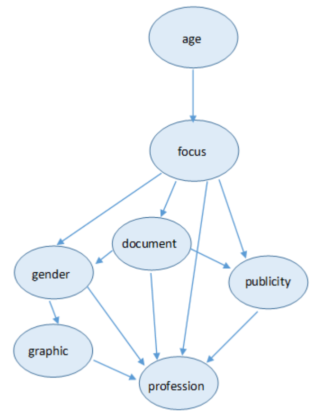

Fig. 2. The structure of the Bayesian network

structural transformation converts the original graph into a junction tree; second, a computational phase executes potential initialization, global information propagation via inter-clique message passing, and final computation of marginal posterior distributions [25].

Traditional Bayesian networks typically represent observations as deterministic evidence, where an observed node is assigned a single specific state with probability 1, while all other states receive probability 0. However, this representation proves insufficient for handling uncertain observations that naturally arise from many real-world measurement processes, where evidence reflects a probability distribution across multiple possible states.

Our approach to user preference modeling incorporates multiple contextual variables as evidence: profession, focus, age, and gender. A crucial methodological innovation involves the discretization of continuous variables like age through fuzzy partitioning techniques for seamless integration into the Bayesian network framework.

Probabilistic observations are generated through fuzzy discretization of continuous variables, grounded in the mathematical formalism of fuzzy partitions. Formally, for a domain Ω, a fuzzy partition consists of fuzzy states $f_1, ..., f_p$ satisfying:

$$\forall x \in \Omega, \sum_{i=1}^{p} \mu_{f_i}(x) = 1, \quad (2)$$

where $\mu_{f_i} : \Omega \mapsto [0,1]$ represents the membership function for state f_i. This framework replaces traditional crisp intervals with overlapping fuzzy states.

An observation $X = x$ is thus transformed into a probability distribution for the discretized variable:

$$R(X_d) = (\mu_{f_1}(x), ..., \mu_{f_p}(x)). \tag{3}$$

Figure 1 illustrates the information extraction process from continuous variable observations. A soft finding for variable X in the Bayesian network corresponds to the probability distribution R(X), obtained by applying fuzzy discretization to the continuous measurement.

The inherent challenge of reasoning with uncertain evidence within Bayesian networks has stimulated the development of diverse algorithmic approaches. These methods often integrate established probabilistic frameworks, including standard Bayesian inference, Pearl's virtual evidence formulation, Jeffrey's rule of probability kinematics, and the Iterative Proportional Fitting Procedure (IPFP) [22, 30, 14].

While effective for small-scale applications, IPFP's reliance on iterative updates to the entire joint probability distribution presents significant scalability limitations for larger networks. In our proposed methodology, evidence propagation is implemented using the Junction Tree algorithm enhanced with probabilistic evidence handling. This algorithm was selected for its capacity to provide exact inference results and its demonstrated computational efficiency in networks with moderate node counts. The comprehensive evidence propagation procedure is formally detailed in Algorithm 1.

Algorithm 1 Algorithm Steps with Fixed Probabilistic Evidence

```
For each state i of the observed node, we perform a classical inference (junction tree) by observing state i.
    For every node N in the Bayesian network:
        For every state j of node N:
            State j =∑ Probability of state j derived from the inference i * degree of
                membership of the observed value in state i.
        End for
    End for
End for
```

4 Evaluation Metrics of an Adaptive User Interface Based on Fixed Probabilistic Evidence

Once the Bayesian network has been established (Figure 3), it can then be utilized to perform inference and draw conclusions about the modeled scenario.

We assessed the usability of our adaptive interface by examining identical usage scenarios across two Bayesian network models, BN1 and BN2. Both models utilized the same discretization scheme, dividing the Age variable into three categories and the Profession variable into two categories. The primary difference between the networks lay in the type of evidence processed: BN1 was restricted to handling hard evidence for these variables, while BN2 was specifically designed to accept and propagate fixed probabilistic evidence.

The study included 286 participants who interacted with both interface types. For each scenario, participants completed surveys, and we compared their self-reported experiences with actual interaction outcomes. The same group of participants provided both behavioral interaction data and subjective survey responses. This section evaluates the performance of our proposed framework through classification tasks using these two distinct Bayesian network configurations.

To assess the effectiveness of our adaptive interface, we conducted a comparative analysis against a conventional fixed interface. The evaluation focuses on the model's predictive capability regarding user preferences.

For this purpose, we employ the Mean Absolute Error (MAE), a widely adopted statistical metric for assessing prediction accuracy in recommender systems research.

The MAE quantifies the average absolute deviation between predicted preferences and actual user ratings.

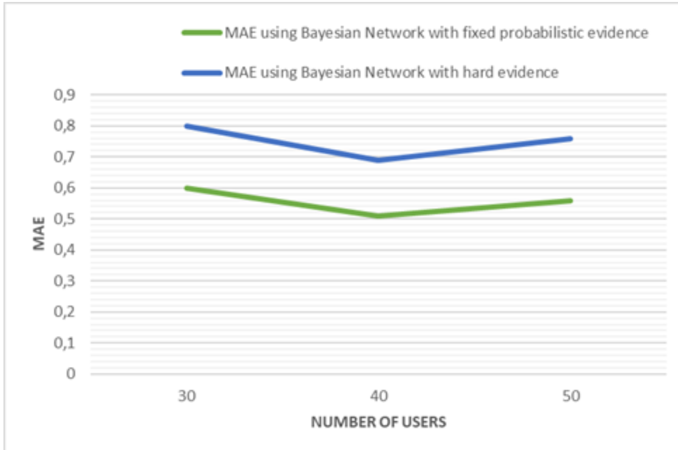

Fig. 3. MAE for the object graphic

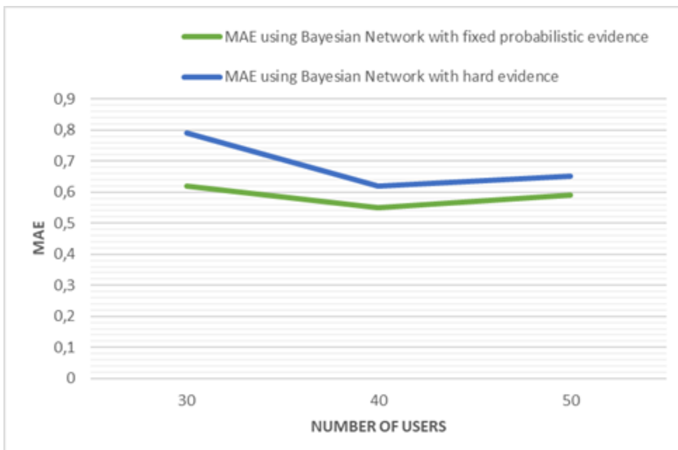

Fig. 4. MAE for the object publicity

Formally, given a test set containing n items, where each item i has an actual user preference a and a_i corresponding predicted preference p_i, the MAE is calculated as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |a_i - p_i| \,. \tag{4}$$

This metric provides a straightforward interpretation of the average prediction error, where lower values indicate higher recommendation accuracy. T

his section presents the experimental results of our adaptive user interface evaluation.

The primary objective of this experimentation is to validate the effectiveness of our proposed approach. We assessed prediction accuracy across different user group sizes for three content types: graphics, documents, and publicity.

Figure 3 illustrates the comparison of Mean Absolute Error (MAE) between the interface using Bayesian networks with hard evidence and the one using fixed probabilistic evidence.

The results clearly demonstrate that our adaptive interface consistently achieves lower MAE values across all user group sizes for graphic content predictions.

For instance, with a group of 30 participants predicting graphic preferences, the interface using a Bayesian network with fixed probabilistic evidence achieved an MAE of 0.60, compared to 0.8 for the interface using Bayesian networks with hard evidence.

Similarly, for the "publicity" object, Figure 4 shows an MAE of 0.62 for the interface using a Bayesian network with fixed probabilistic evidence, compared to 0.79 for the interface using Bayesian networks with hard evidence.

This represents a statistically significant improvement, confirming the advantage of the Bayesian network approach implemented in our adaptive interface.

5 Conclusion

We propose that fixed probabilistic evidence significantly improves Bayesian user modeling by making its inference capabilities more flexible and expressive. Our empirical evaluation confirms that propagating this evidence leads to more accurate preference inference compared to standard methods.

For even greater effectiveness, we also highlight the importance of robust training using user activity data.

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**Corresponding author is Rebai Rim.*