

A Class of Exponential-Type State Observer Design for the Estimation of the Risk Population on the COVID-19 Pandemic

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Abstract. The aim of this study was to present a class of exponential-type state observer designs for the estimation of the risk population on the COVID-19 pandemic. The observer is based on the SIDUR-type model for the prediction of the risk population in the COVID-19 pandemic, which is one of the popular employed mathematical models. The proposed observer considers as measured variables, the number of diagnosed persons who are infected and detected by test and the number of removed persons who either recover or die after being diagnosed notice that these considered measured signals are commonly reported by the public health systems. A local observability analysis shows the observability characteristics of the considered model. The most important feature of this observation method is the convergence can estimate the number of infected persons who are undetected by the public health systems. Finally, in silico experiments show the satisfactory performance of the proposed methodology and asymptotic stability of the estimation convergence is shown in a Lyapunov framework. The utility of this methodology is that it provides information to support public health decision making during a pandemic, and a strength of the work is that it can as a reference for a new epidemics spread.

Keywords. COVID-19 pandemic, state estimation, SIDUR-type, asymptotic convergence.

1 Introduction

The epidemic of Coronavirus Disease (COVID-19) is a timely memorial of the nature and impact of

arising contagious conditions that come Public Health Emergency of International Concern (PHEIC) [5]. The COVID-19 epidemic takes variable shapes and forms in how it affects different regions and countries.

In 2022, there were over 500 million cases and over 6 million deaths notified around the globe since the launch of the epidemic [6, 16]. Millions of persons around the world have been infected with COVID-19 in alarming rates, whereby, it was declared a global pandemic and Public Health Emergency of International Concern (PHEIC) by WHO [27].

SARS-CoV-2 virus causes a respiratory infection named Coronavirus disease 19. Scientist across the world have intensified their efforts to develop a vaccine or drug capable of treating COVID-19 [23].

A serious problem is the lack of information for monitoring the COVID-19 pandemic. The above can partially be explained by variations in testing algorithms, capacity for testing, and reporting across different countries.

No country knows the total number of people infected with COVID-19. All we know is the infection status of those who have been tested. Only those who have a lab-verified infection are counted as verified cases Epidemiological studies that divide a population into compartments [14]. To fully understand the nature of the pandemic and

the impact of social distancing measures, one must understand the mathematics behind it.

One of the most relevant mathematical models relating to the spread of a pandemic is the susceptible-infectious-removed (SIR) model and its variants (called compartmental models) [34]. The model predictions overestimated the number of confirmed cases due to the exponential growth, and the discrepancy between the model predictions and actual data could be attributed to the restriction of social activities such as the lockdowns of cities [47].

In the real data, there is a constant trend after a wave in the pandemic passes, which makes the models unpredictable. Therefore, it is a challenging future problem to extend the model to predict to long-term for example months. Now, to try to avoid the lack of information about the population susceptible persons who are prone to the complaint and the number of infected persons who are undetected by the public health systems, several modelling strategies have been developed to give trends on the epidemic evolution. Standard model-grounded strategies lead to the typical model downsides, as parametric identifiability, and high parametric sensibility [21, 37].

Considering that without data on who's infected by the contagion, we have no way of understanding the epidemic. Without this data, we cannot know which countries are doing well, and which are just underreporting cases and deaths. Also, the employment of state observers can be useful to gain estimated data about the missing information on specific population sectors affected by the epidemic [17, 40].

The state observers design is a natural issue in control theory, where the application on ecological systems, natural processes, and technological systems has been widely employed [8, 46]. Then, state observers designed grounded on the sliding-mode theory, the high gain observers, extended Luenberger and Kalman filters, between others have been consecutively applied [8, 19].

In addition, without reliable test data, feedback measurement of the infected, it is highly unlikely to predict the growth rate of any epidemic, making the estimate highly uncertain [35]. For this reason, a state estimation-based robust control method has

been applied in on the H-infinity Kalman Filter for the acute inflammatory can predict without knowing the entire state vector [46]. The extended Kalman filter (EKF) technique has been used as the state observer to predict the likelihood of Ebola disease spread [32]. A state estimation-based strategy has been presented in that only needs the numbers of exposed and infected humans as the feedback signals to control the influenza epidemic [25, 38].

Also, to try to ameliorate the model-grounded estimation strategies, the use of the state observers plays an important part, similar that a state bystander can be conceptualized for a real-time adaptive model, which improves the variable's prognostications by including measured information of official health authorities (i.e. the number of diagnosed persons who are infected and detected for predicted the number of susceptible persons who are prone to the disease and the number of infected persons who are undetected) [32, 39].

Although various estimated strategies have been proposed to address the COVID-19 pandemic using observer's state, many of these suffer from severe drawbacks: for example, an observability analysis, a simple structure estimation proposal with real data, and easy implementation, finally estimate the number of infected people who are undetected by the public health systems.

Motivated by this gap, this paper presents the proposed a novel state observer approach to estimate number of infected people who are undetected by the public health authority. The observer considers as measured variables, the number of diagnosed persons who are infected detected by test and the number of removed persons who either recover or die after being diagnosed. The observability analysis and the associated algorithms for the observer design it possible to accurately estimate the target states of a large-scale using minimal information. While the majority of estimation methods are based on stochastic process, this new approach is deterministic.

The method is efficient in the sense that the epidemiological data is injected directly into the model once we found a constant observer gain. In epidemics such as the COVID-19 pandemic,

Table 1. Values of parameters used in the SIDUR model

Parameter	Values	Description
b_1	0.3708	Infection rate
b_2	0.0707	Infection rate
b_3	0.3717	Infection rate
δ	0.1589	Recovery
γ	0.0499	Removal rate
μ	0.5000	Testing rate

we demonstrate that the proposed functional observer can infer the can estimate the number of susceptible persons who are prone to the disease and the number of infected persons who are undetected.

2 Materials and Methods

2.1 Covid-19 Propagation Mathematical Model

The model SIR has been used in several studies analyzing the spread of COVID-19 [19, 32, 35, 38] and [2]. These models help us understand how COVID-19 spreads, predict the regional peaks of the pandemic, and understand the impact of various quarantine measures [7, 18, 22]. All the epidemiological models, which we will discuss in this paper, are based on analyzing systems of differential equations. Differential equations focus on the rate of change of a variable, or group of variables as time passes. In this work, presented the fine model, which describes, the covid-19 propagation on a mortal population, the model lies on the family of models named SIDUR, susceptible (S), undiagnosed infected (I), diagnosed infected (D), unidentified recovered (U) and linked removed (R) [33].

We consider a five variable model included the population groups above mentioned. We define $x_S(t)$ as the number of susceptible people, who are prone to the complaint; $x_I(t)$ is the number of infected people who are undetected by the health authority; $x_D(t)$ is related to the number of people diagnosed, who are detected as infected by a test; $x_U(t)$ identify the recovered people which is unidentified, without opinion and eventually $x_R(t)$ represents the number of persons who either

recovered or die after being diagnosed. As a supposition is considered the number of persons of the population under study remains constant N .

The fine model is represented by the following set of ordinary differential equations:

$$\begin{aligned}
 \frac{dx_S}{dt} &= -\frac{\alpha x_S x_I}{N}, \\
 \frac{dx_I}{dt} &= -\frac{b_1 x_S x_I}{N} \frac{\mu x_I}{x_T} - \delta x_I, \\
 \frac{dx_D}{dt} &= \frac{\mu x_I}{x_T} - \gamma x_D, \\
 \frac{dx_U}{dt} &= \delta x_I, \\
 \frac{dx_R}{dt} &= \gamma x_D.
 \end{aligned} \tag{1}$$

Here the population remains constant during the evolution of epidemic:

$$N = x_S(t) + x_I(t) + x_D(t) + x_U(t) + x_R(t),$$

where N stands for the total population and:

$$x_T = x_I(t) + (1 - 0.75)(x_S(t) + x_U(t)),$$

under the following set of parameters described in Table 1.

2.2 Observability of the Covid-19 Propagation Model

One of the various concepts accustomed analyzes dynamical systems and networks is observability. On top of things theory, the cradle of this idea, observability is expounded to the power of reconstructing the state of the system from a limited set of measured variables in finite time. A somewhat relaxed version of this definition of observability and which is applicable to networks is thought of as structural observability and will be assessed with a graph [24].

The preceding problem to observer design is to analyze the observability conditions of the nonlinear systems under study. For linear systems, classical observability index as observability matrix and observability Grammian for observability analysis and the estimator design has been extensively studied and has proven extremely useful, especially for online monitoring and control applications such as observer-based control design [29].

2.2.1 Local Observability under Linear Approach

A general form for a mathematical representation of a dynamic system is as follows:

$$\dot{x} \equiv \frac{dx}{dt} = f(x), \quad (2)$$

where $x \in \mathbb{R}^n$ is the state variable vector, $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a nonlinear function, where $f(x) \subset \Sigma \in C^\infty$ and Σ is a bounded compact set.

2.2.2 Linearization via Taylor Series

To linearize general nonlinear systems, we will use the Taylor Series expansion of functions. Consider a function $f(x)$ of a single variable x , and suppose that $f(x^*)$ is a point such that $f(x) = 0$. In this case, the point $f(x)$ is called an equilibrium point of the dynamic system $\dot{x} = f(x)$, since we have $\dot{x} = 0$ when $x = x^*$ such that the system reaches an equilibrium point.

In the study of dynamical systems, the linearization theorem is a theorem about the local behavior of dynamical systems about a hyperbolic equilibrium point. It asserts that linearization sample of the original nonlinear system is effective in predicting qualitative dynamics of behavior.

The Hartman-Grobman theorem states that the performance of a dynamical system in a domain near a hyperbolic equilibrium point is equal, as the behavior of its linearization near this equilibrium point.

The (generalized) eigenvectors akin to eigenvalues with positive real parts form the unstable Eigen-space. However, if the equilibrium has eigenvalues whose real part is zero, then the corresponding (generalized) eigenvectors form the middle Eigen-space. Going beyond the linearization, once we account for perturbations by nonlinearity or forcing within the phase space, the middle Eigen-space deforms to the nearby manifold.

If the eigenvalues are precisely zero, instead of just real-part being zero, then the corresponding Eigen-space more specifically gives rise to a slow manifold. The behavior on the middle (slow) manifold is mostly not determined by the

linearization and thus is also difficult to construct [41].

Let us consider the following linear system representation of the system Eq. (1):

$$\dot{x} = Ax, \quad (3)$$

$$y = Cx, \quad (4)$$

Where A is defined as:

$$A(\bar{x}) = \left[\frac{\partial f}{\partial x} \right]_{x=\bar{x}}.$$

Now, from a general framework, let us to consider the following, be the finite sets

$Y = [y, y', y'', y''', \dots, y^{n-1}]^T \in \mathbb{R}^m$ and $X = [x_1, x_2, x_3, x_4 \dots x_n]^T \in \mathbb{R}^n$, with $m \leq n$ which is related with the system output vector and finite time derivatives, therefore it is possible to construct the following linear dynamic system:

$$\begin{bmatrix} y \\ y' \\ y'' \\ \vdots \\ y^{n-1} \end{bmatrix} = \begin{bmatrix} Cx \\ CAx \\ CA^2x \\ \vdots \\ CA^{n-1}x \end{bmatrix}. \quad (5)$$

Or in vector form:

$$Y = OX,$$

where:

$$O = [C, CA, CA^2, CA^3, \dots, CA^{n-1}]^T. \quad (6)$$

As can be seen, if the state vector can be determinate, the matrix O (named as the observability matrix) must be invertible (full rank) to obtain:

$$X = O^{-1}Y.$$

Such that, the state vector X is observable in respect to the measurable output Y .

The local observability analysis is condensed as follows: A continuous-time linear (or linearized) system is observable if and only if $rank(O) = n$, where n is the order of the system [29].

2.2.3 Observability Calculation

Let us consider a linearized version of the model Eq. (1) around the equilibrium point $x_{ss} = [884 \ 0 \ 0 \ 17352 \ 17069]$, in this equilibrium point the corresponding Jacobian matrix is given by:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} 0 & -0.009 & 0 & 0 & 0 \\ 0 & -0.15 & 0 & 0 & 0 \\ 0 & 0 & -0.045 & 0 & 0 \\ 0 & 0.159 & 0 & 0 & 0 \\ 0 & 0 & 0.045 & 0 & 0 \end{bmatrix}.$$

The eigenvalues of the above Jacobian matrix are:

$$\lambda = [0 \ 0 \ -0.1500 \ 0 \ -0.0450]^T.$$

From the above mentioned, the dearth of knowledge within the linearized version of the system Eq. (1), will have a consequence that the entire observability of the system won't be found. Now, it's vital to pick the corresponding output measurements to the system Eq. (1), such the observability properties of the system depend strongly on this selection.

As natural the choice of the output measurements y must be related to the variables which are easier to determinate, during this case, this corresponds to $x_D(t)$ which is said to be the amount of diagnosed people, who are detected as infected by a test and $x_R(t)$.

It represents the number of persons who either recovered or die after being diagnosed, as will be noticed these two variables may be measured by the health system authority with accuracy.

From the above, as a first approach, the variable $x_D(t)$ is selected as measured output, then in accordance with the notation of Eqs. (3) and (4):

$$C = [0 \ 0 \ 1 \ 0 \ 0].$$

From Eq. (6), we obtain the observability matrix presented in Table 2 as Matrix C .

For the above observability matrix, the corresponding rank is $\text{rank}(O) = 2$. Now, considering as measured output $x_R(t)$, the matrix C is defined as:

$$C = [0 \ 0 \ 0 \ 0 \ 1].$$

For this measured output, the corresponding observability matrix is given by matrix in Table 2 as Matrix O . The corresponding matrix rank is $\text{Rank}(O) = 3$.

Finally, we can consider the combination of both output measurements mentioned above. This led to a matrix C as follows:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Here, the corresponding observability matrix is given by

$$O = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & -0.0499 & 0 & 0.0499 \\ 0 & 0 & 0.0025 & 0 & -0.0025 \\ -0.0002 & -0.0034 & 0 & 0.0036 & 0 \\ 0 & 0 & -0.0001 & 0 & 0.0001 \end{bmatrix}^T$$

where the related rank is $\text{rank}(O) = 3$.

Analyzing the above results, the selection of the measured output $x_D(t)$ can be discarded, because this selection produces an observable subspace with dimension 2 the selection of $x_R(t)$ generate an observable subspace of dimension 3 and finally, the combination of $x_D(t)$ and $x_R(t)$ as measured outputs, produce an observable subspace with dimension 3.

Then, the variable $x_R(t)$ is selected as measured output, such that it generates the higher dimension of the observable subspace and fortunately, the variables $x_S(t)$ which is the number of susceptible people, who are prone to the disease and $x_I(t)$ as the number of infected people who are undetected by the health authority is observable, this is very important because these two variables are unmeasured and very important from the epidemic point of view.

Table 2. Matrices in calculations

$$C = \begin{bmatrix} 0 & 0.0001 & -0.000 & -0.0002 & -0.0034 & 0.0000 & 0.0036 & -0.000 & -0.000 \\ 1 & -0.0499 & 0.0025 & -0.0000 & -0.0000 & -0.0001 & 0.0000 & 0.0001 & 0.0000 \end{bmatrix}^T$$

$$O = \begin{bmatrix} 0 & 0 & 0.0000 & -0.0002 & -0.0034 & 0.0000 & 0.0036 & -0.0000 & 0.0000 \\ 0 & 0.0499 & -0.0025 & -0.0000 & -0.0000 & -0.0001 & 0.0000 & 0.0001 & -0.0000 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Observer Design

Proof. The following nonlinear system is an asymptotic observer to the state vector of the system Eq. (1):

$$\dot{\hat{x}} = f(\hat{x}) + (g_1 \exp(e) + g_2)e, \quad (8)$$

where g_1 and g_2 are the observer's gains. The vector and the estimation error is defined as [41]:

$$e = y - \hat{y} = Cx - C\hat{x}.$$

Let us consider the observation error dynamic from equations (2) and (8):

$$\frac{de}{dt} \equiv \dot{e} = f(x) - f(\hat{x}) - g_1(\exp(e) + g_2)e. \quad (9)$$

Notice that since the functions $f(x)$ is a class C^1 with respect to x , which is globally Lipschitz continuous.

Assumptions

We are making the following assumptions.

$$A1. \quad \|f(x) - f(\hat{x})\| \leq F\|x - \hat{x}\|,$$

where F is the named Lipschitz constant, defined as:

$$F = \max_x \|J(x_{ss})\| = \max_{1 \leq i \leq m} \sum_{j=1}^n |J_{ij}|. \quad (10)$$

Now, applying the Cauchy-Schwartz inequality to Eq. (9):

$$\|\dot{e}\| \leq F\|e\| - \|g_1 \exp(e) + g_2\|\|e\|.$$

Note that:

$$\|\exp(e)\| \geq 0.$$

Let us propose the following quadratic form of a Lyapunov function:

$$V = \frac{1}{2}e^2.$$

Note that this scalar function is a positive definite function. The time derivative of the scalar function V is defined as:

$$\dot{V} = e\dot{e}. \quad (11)$$

Substituting Eq. (9) on equation (11):

$$\dot{V} = e(f(x) - f(\hat{x}) - (g_1 \exp(e) + g_2)e).$$

Now, considering the inequality Eq. (8):

$$\dot{V} \leq (F - g_2)\|e\|^2 - g_1\|\exp(e)\|\|e\|^2.$$

If g_2 is selected such that $F < g_2$, then:

$$\dot{V} \leq ((F - g_2) - g_1\|\exp(e)\|)\|e\|^2 \leq 0. \quad (12)$$

□

Then Lyapunov can conclude arguments, that the proposed observer is stable. The Lyapunov method has been applied in many research areas to analyze the stability of dynamic systems Eq. (1). To prove that a system is stable in the sense of Lyapunov, a positive definite function of the state variables is shown to decrease along system trajectories Eq. (8).

Indeed, the stability of the estimation error of observers for noise-free systems can be addressed by using Lyapunov functions [1, 15].

3 Results and Discussion

Numerical simulations were carried out in order to show the performance of the proposed methodology in a PC with Intel® Core™ i7-10870H processor and employing the 23s Matlab™ library as ordinary differential equations solver.

The initial conditions of the system Eq. (1) are $x_0 = [11000 \ 5459 \ 7500 \ 825 \ 2500]$ respectively for each one of the state variables; on the other hand, the initial conditions of the proposed observer are $x_0 = [9250 \ 6000 \ 7950 \ 875 \ 2200]$.

Now, for nonlinear systems, the tuning of parameters for observers and controllers is a challenge and the published results correspond to specific examples [30, 36, 43] the vector of the observer's gain is $g_1 = [10 \ -10 \ 10 \ 10 \ 10]^T$ and $g_2 = [0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.2]^T$.

The selection of the parameters is done via the required results of the Lyapunov stability analysis to induce that the time derivative of the Lyapunov function be negative semi-definite, see Eq. (12).

The Lipschitz constant is evaluated based on Eqs. (7) and (10), then, $F = 0.159$. In accordance with the observability analysis, it is considered the variable $x_R(t)$ as the measured output for the observer implementation, such that it is routine measured data from the health authority system, as above. The corresponding observable variables are $x_I(t)$, $x_U(t)$ and $x_R(t)$. Here is very important to remark that $x_I(t)$, the number of infected people who are undetected by the health authority is a piece of key information to know the pandemic evolution.

Figure 2 shows the observer's performance for an unobservable state variable $x_R(t)$, as expected, the estimation result is poor, because the observed state does not follow the trajectory of the designed real trajectory adequately.

Now, figure 3 is related to the estimation of the key variable $x_I(t)$, here can be noticed a satisfactory performance of the estimated trajectory performed by the proposed observer, remembering that this state variable is observable. In Figure 4 is seen the evolution of another unobservable state variable $x_R(t)$, in

consequence, the estimation performance is also unsatisfactory.

Figure 5 presents a phase portrait with the real trajectory in the 3D x_S , x_I , x_D , state space, together with the estimate trajectory generate by the proposed observer, note that the mismatch between both trajectories is due the unobservability characteristic of the state variables x_S and x_D , which generate a poor estimation prediction.

Figure 6 shows the evolution of the routine measured data, which were considered for measured outputs $x_D(t)$ and $x_R(t)$, and finally, a comparison between the proposed observer and a standard extended Luenberger observer (ELO) is done [13], the ELO is implemented under the same initial conditions and vector's gain, similar with the sliding-mode [12] and the high order sliding-mode observers [10, 45].

Figure 7 shows the evolution of a performance index $PI = \int_0^t \text{abs}(e(\tau))dt$, where is the estimation error, as can be noticed the ELO presents a very poor performance with higher values of the performance index.

Infectious diseases are a threat to the health of the world population. One of the easiest ways for individuals to reduce the risk of infection during an epidemic is to decrease their rate of contact with infected individuals. However, the value of these actions depends on how the epidemic progresses.

Few cases analyses indicate how changes in behavior will change the epidemic. Consequently, estimates based on state observers are a viable alternative since they improve the performance of the mathematical models named SIDUR, allowing the generation of information to know variables that are not easy to quantify during the evolution of the pandemic.

It can be observed from the performance index (estimation errors) that the proposed observer estimates these dynamics reasonably accurate. Proposed observer is compared with the standard extended Luenberger.

In addition, Table 3 shows the sales of our proposal as well as a different and simple approach in relation to recently published research. It can be observed that the estimation results are virtually different respectively. We should note, however,

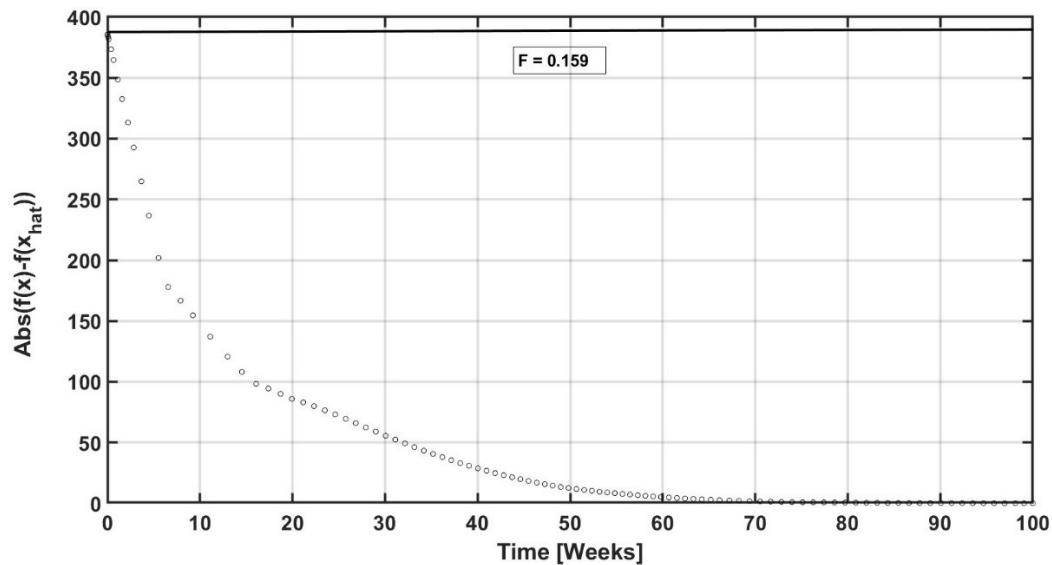


Fig. 1. Numerical validation of the Lipschitz conditions of the SIDUR-type model.

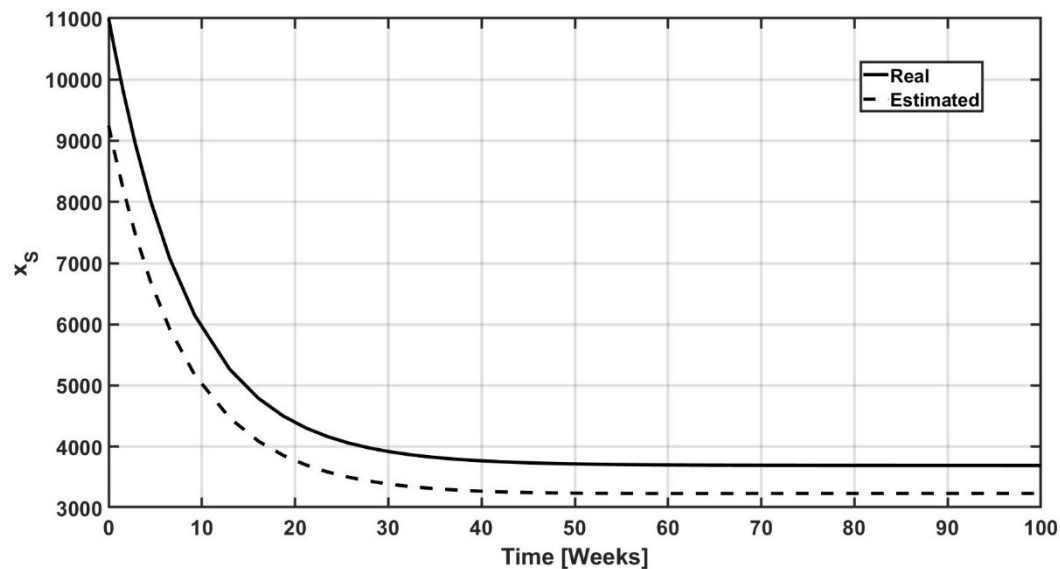


Fig. 2. Time evolution of the unobservable state $x_S(t)$

that these results are obtained after a lengthy process of trial and error when determining the observer gain vector [30].

Having said that, the main advantage of this approach is its stability and efficiency compared

to standard extended Luenberger, and others approach that require calculation of inverse matrices. The observer results from can act as a benchmark while devising COVID 19 control strategies [3, 4, 9, 11, 20, 31, 42, 44, 45].

Table 3. Comparison of state observer design algorithms for COVID-19 pandemic

Methodology implemented	Available and reconstructed variables	Advantages or Disadvantages	Ref.
SEIR-based model is proposed considering dynamic feedback estimation. State and parameter estimations are performed on classical methods of estimation. Constrained extended Kalman filter (CEKF), CEKF and smoother (CEKF & S), and moving horizon estimator (MHE).	The parameters estimation was based on vaccine efficacy studies regarding transmissibility, severity of the disease, and lethality. Unmeasurable: Susceptible, Exposed, Infected, and Ailed. Unmeasured: Quarantined, Recognized, and Healed detected. Measured: Treated, Healed with treatment, and Deceased.	It comprehends estimates of unknown properties based on available measures while filtering them to reduce the noise Effects Disadvantages: Assumption, observability of state variables; Implementation in real time is not simple they do not guarantee realistic estimations; No estimated available data of confirmed cases or number of infected persons.	[4]
Luenberger-like state observer Observer nonlinear design method is based on positive definite matrices.	The available online measurements data provided are the global infected and dead population. Monitoring the COVID-19 epidemic in a prescribed host region.	Analysis of the proposed Linear matrix inequalities (LMI) which guarantees a robust convergence and detectability. Disadvantages: LMI based on constraints which complicates the real time prediction and estimation.	[20]
A sliding window algorithm, based on metapopulation SEIR (mSEIR) model for the parameter estimation and the prediction.	Parameter estimation in the model and further predict the infected populations to obtain the trend.	Implemented artificial intelligence techniques and the network science tools. Disadvantages: it fails to predict the long-term evolution.	[42]
Observer-based predictor for a time t is the number of infected susceptible-infectious-recovered people and predicted susceptible. model with delays.	The only information available at time t is the number of infected people and predicted susceptible.	The predictor is evaluated using real data from a COVID-19 epidemic.	[11]
Predicted based in two basic (SIR) and extended (SQAIR) epidemic models was evaluated. In both strategies, unknown parameters of the model were estimated using real data and the least-squares method.	Predicted the incorporating the impact of asymptomatic people, in addition to as like the transmission rate, recovery rate, and mortality rate, are estimated.	The model got more realistic to consider a quarantined people as state variable. Disadvantages: The compartmental model no considered limitations regarding the assumptions. In addition, all numerical results are dependent on the initial condition.	[9]
Super Twisting Algorithm (The 2d-order sliding-mode) and Least Squares Estimation (LSE) algorithm.	Sub-population susceptible to acquire the disease, sub-population that has become infected, sub-population that has recovered from infection.	The convergence is finite time, and robustness allows us to reconstruct non-measurable variables. Disadvantages: assume observability of state variables.	[31]
Exponential-type state observer-based estimation.	The number of infected people who are undetected by the health authority is observable. Available variables are the number of diagnosed persons who are infected and detected by test and the number of removed persons who either recover or die after being diagnosed notice.	The implementation of the proposed observer is simple to implement because it has fewer parameters for tuning. Observability analysis. A simple proposed observers is preferable when the objective is to devise decision strategies to short-term.	In this work

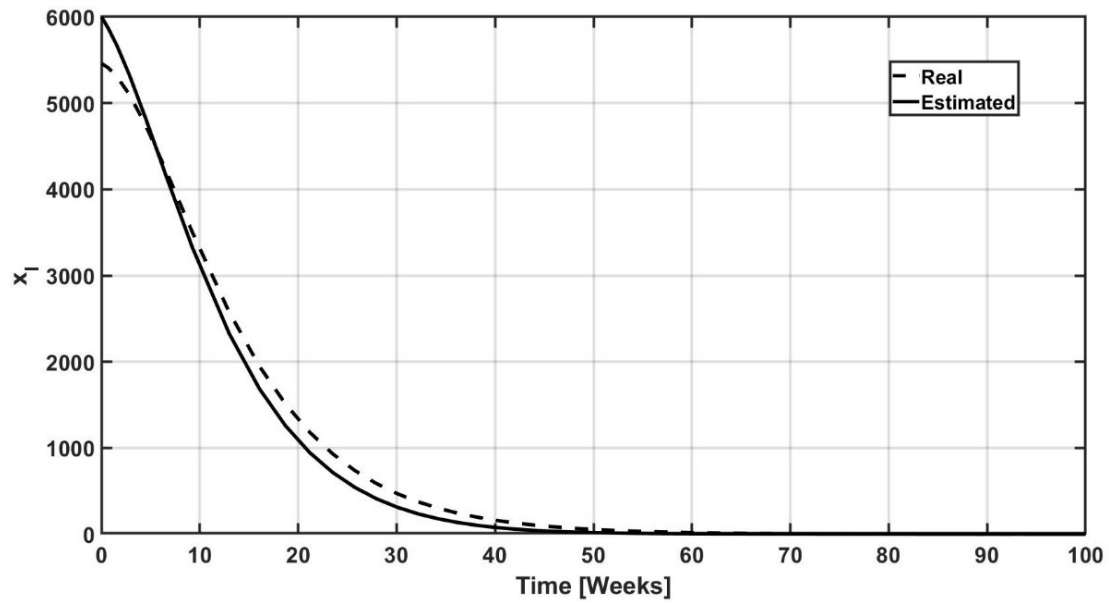


Fig. 3. Time evolution of the observable state $x_I(t)$ under the proposed observer

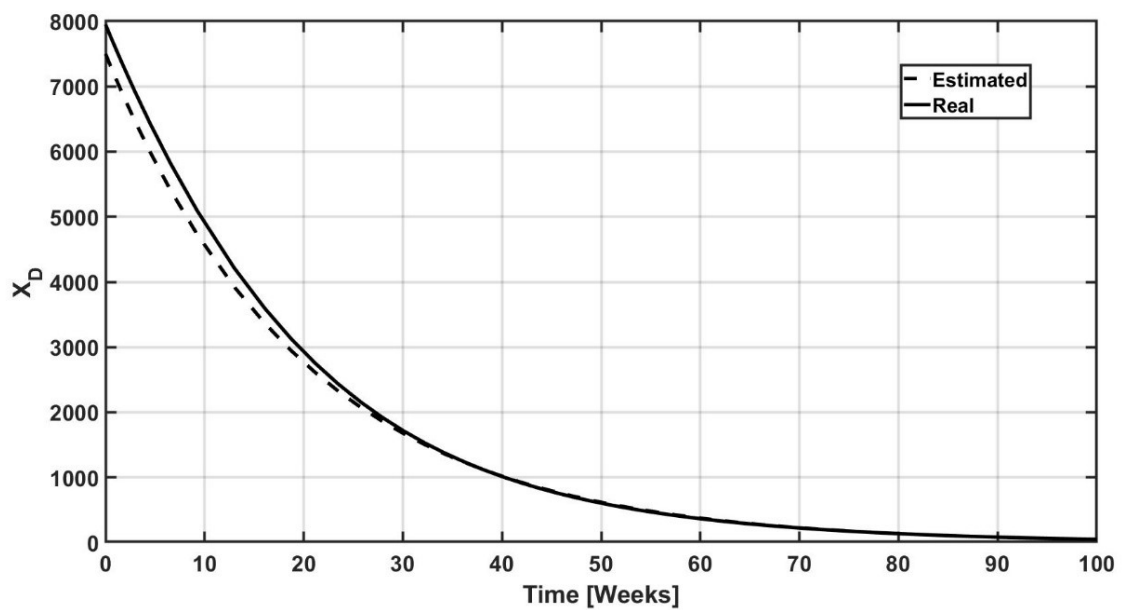


Fig. 4. Time evolution of the unobservable state $x_D(t)$ under the proposed observer

Therefore, as a future prospect, this allows us to predict the unmeasured states of the model.

Nonetheless, the proposed method allowed the study of overall dynamics.

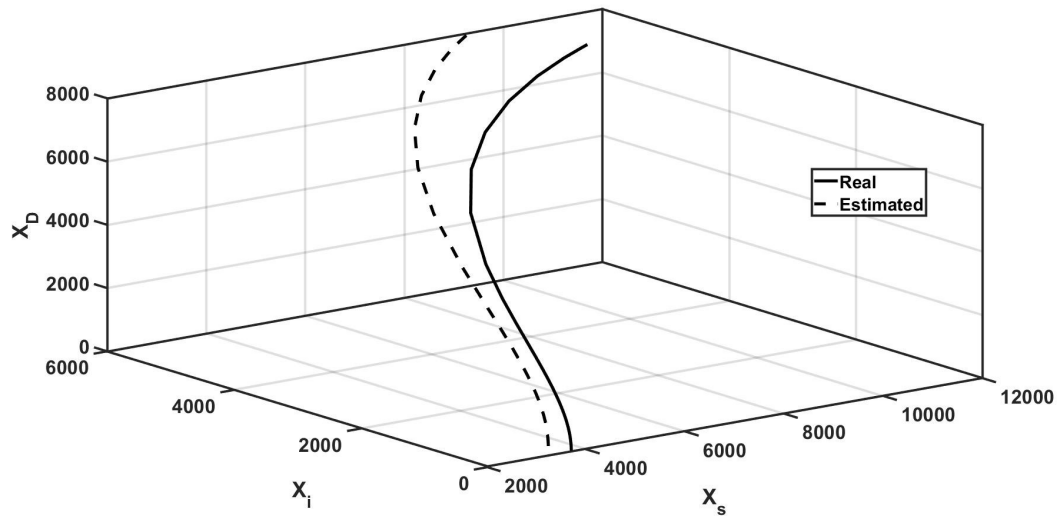


Fig. 5. Phase portrait of the real trajectory and the estimate trajectory for unobservable variables and the observable variable

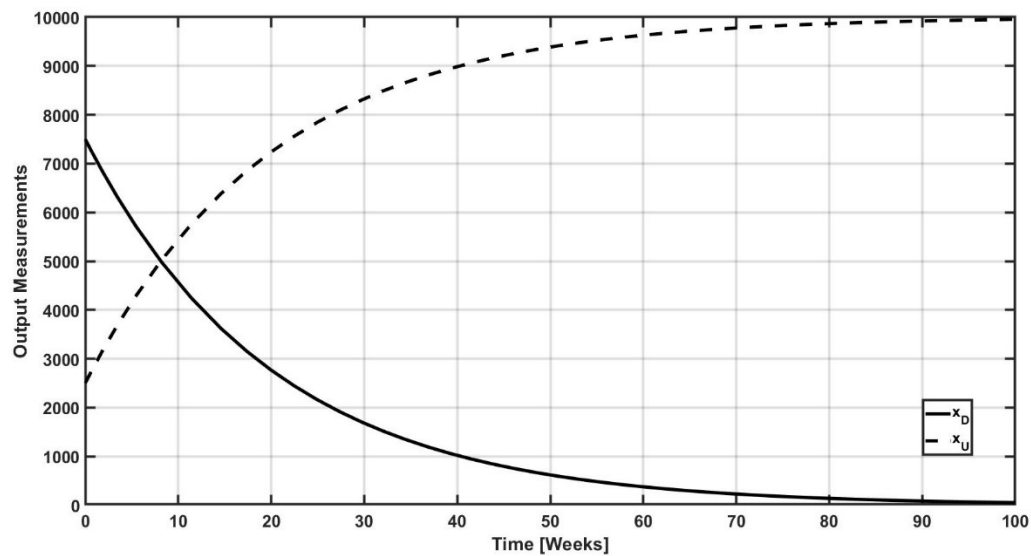


Fig. 6. Time evolution of the considered measured data

Impact

The utility of this methodology is that it provides information to support public health decision making during a pandemic, and a strength of

the work is that it can as a reference for a new epidemics spread.

In particular, we emphasize the opportunities to exploit the potential of estimation based on models that could be used worldwide to support

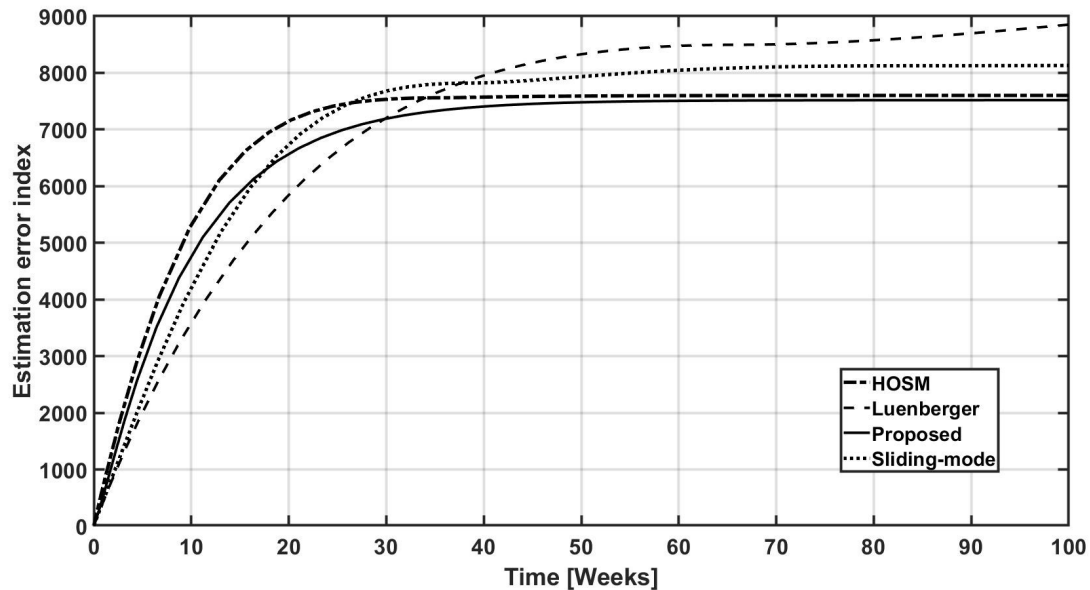


Fig. 7. Comparative evolution of the performance index of the proposed observer and other observer structures

policy making to assist in decision making at a strategic, tactical and operational level that provide information in real time on status of behavior pandemic that could be beneficial to healthcare organizations.

In epidemics such as the COVID-19 pandemic, we demonstrate that the proposed functional observer can infer the can estimate the number of susceptible persons who are prone to the disease and the number of infected persons who are undetected.

In addition, a class of nonlinear observer with an exponential-type structure is proposed with adequate estimation results. The convergence of the proposed observer is done via the Lyapunov framework [26, 28, 48].

Limitations

The prediction results are based on the model, however this is an area of opportunity because the model can be substituted and predict the behavior of another public health propagation problem. The accuracy of our data must be considered.

Thus, even if we can be confident in providing predictions for medical care organizations we urge the reader to run our simulations with their own data or model, which can be achieved using the proposed methodology.

The model and observer are quick to run with a focus on accessibility so that a user can rapidly change the input to suit their data or model.

Thus, our numbers are a lower bound on the likely impact of new cases.

Furthermore, this work does not consider secondary cases, where number of infected people are infected by other types of association such as diabetes mellitus and arterial hypertension as risk factors for mortality in patients with this coronavirus.

4 Conclusions

In this work is presented the observability analysis of a SIDUR-type model, considering as measured outputs two variables routine obtained by the health authority system $x_U(t)$ and $x_R(t)$, where, as an important result, the state variable $x_I(t)$, which represents the number of infected

people who are undetected by the health authority is observable.

By the observability results, $x_R(t)$ is chosen as the measured output to implement the proposed state observer, here the dimension of the observable subspace is 3.

In addition, a class of nonlinear observer with an exponential-type structure is proposed with adequate estimation results. The convergence of the proposed observer is done via the Lyapunov framework.

The proposed methodology is compared with a standard extended Luenberger, sliding-mode and high order sliding-mode observers via a performance index of the estimation error, here the proposed observer has the better performance. In particular, we emphasize the opportunities to exploit the potential of estimation to assist in decision making in pandemics that could be beneficial to healthcare organizations.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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