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Logistics Application for the Capacitated Facility Plant Location Model for Distribution Centers

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Abstract. The location and allocation facilities problems are frequently used in different areas within organizations to improve the logistics of process flow, in order to achieve their strategic objectives in the short, medium and long term. A proper analysis of facility location problems allows companies to make efficient decisions to minimize distribution costs, and streamline operational times, and as a result, companies evolve and rebound towards growth. This work focuses on the facility allocation and location problem using the Capacitated Facility Plant Location (CFPL) model to solve a distribution problem and locate a supply center. It is applied for an instance of a set of 25 cities in a geographic area of Mexico, particularly the State of Puebla. Because the CFPL is a mixed integer programming problem, the Branch and Bound method was used through the Lingo software to have a distribution logistics network system.

Keywords. Branch and bound, CFPL, location-allocation, logistics.

1 Introduction

The location of facilities plus distribution models within the supply chain are necessary factors when managing modern logistics operations. All these elements are related when managing and operating the strategic activities of the facilities, for example: the fact of having a warehouse located at a certain point to supply different sites will influence costs, times, and distances, in addition to the design of routes for the transfer of products or services if applicable.

Taking these elements into account, it can be stated that the procedures for locating facilities are complex, since, in the configuration of various aspects to locate, decisions must be made about the optimal site through a study of critical factors such as: the geographical area, work environment, available resources, accessibility, transportation, costs, payment of fees, among others.

All this is to locate warehouses, logistics hubs, production plants, distribution centers or cross docks.

For some researchers [1, 2], the location of facilities usually focuses on the optimization of supply chains around the world, which allows logistics operating costs to be minimized, for logistics experts [3] such as party logistics, considerations of workforce availability criteria, transportation-related costs, government regulations and restrictions plus the political climate generate competitive advantages when locating and designating facilities.

In addition to the previous elements, there are other perspectives focused on location problems such as the capacitated plant location problem [4, 5] that have a high influence on logistics issues since it will have a consequence according to the restriction of the plant capacity that will directly impact meeting the demand of customers located in a specific geographic area.

To provide an optimal solution to the problems of location and allocation of facilities, different techniques have been implemented such as exact methods, heuristics, and metaheuristics [6, 7].

This work addresses the CFPL [4,5] modeled as an integer programming formulation that is solved by means of Branch and Bound method [8] with Lingo.

The CFPL is a location-allocation type model, in the case of the binary integer programming approach there are two decision variables: the first is location to determine the location points around which users are grouped, the second is an assignment variable where users are assigned to the location point forming clusters of users around a distribution center, mathematically partitions are formed of a subset of the natural numbers and although this problem belongs to the NP-hard class, that is, there is no algorithm that solves it in polynomial time, such that only low-scale cases can be solved by means of integer programming methods such as Branch and Bound [10].

In this document the Branch and Bound method is used because it is a binary integer programming model that was implemented in Lingo, although there are other specific techniques for this class of problems of high computational complexity such as: heuristics based on Lagrangian relaxation [18].

For this reason, commercial software such as Lingo was used, obtaining a global optimal solution for an instance of 25 nodes.

For this reason, commercial software such as Lingo was used, obtaining a global optimal solution for an instance of 25 nodes. A hypothetical case is presented using 25 cities as a test for the distribution of various products, the central point is to locate new distribution centers that satisfy the demand of customers located in the same cities.

The work is structured as follows: introduction as section 1. Section 2 deals with the preliminaries of this manuscript. Section 3 focuses on the general aspects related to the methodology of CFPL. Section 4 presents the case of study. Section 5 deals with the results and discussion and finally the conclusions are presented in section 6.

2 Preliminaries

Plant location problems have been used over time to solve customer demands considering the efficient use of their resources [13, 14]. This task has been arduous within the investigation; however, new ways of proceeding have been found to solve them.

The objective of the problem is basically to provide access to clients with the available resources, considering the elements of distance, costs, and time.

The p-Median method (PMP) is frequently used to resolve this type of situation [9] and consists of locating p facilities within a network to supply customer demand. The objective function is considered to minimize the distance or time. The traditional way to solve the problem is to give a set of n demand points and another set of n supply points that minimize the total distance of the demand and supply points.

New applications that include the rectilinear and positional Euclidean versions of the problem as an NP-hard, considering the measurement of the distance in this case impacts the complexity of the problem as well as the solution [10]. Other works [11, 12] explore the p-mendian to republish the distances and locate postal service companies, this to group clients to the closest origin.

The Floyd-Warshall algorithm and the Set Covering Problem (SCP) are two distinct but important topics in the field of computer science, but specifically in the facility location problems. The Floyd-Warshall algorithm is an algorithm for finding the shortest paths between all pairs of nodes in a weighted graph, while the SCP is a combinatorial optimization problem that involves finding the minimum cost subset of sets that covers all the elements in each graph.

Although these two topics may seem unrelated at first sight, there are several works that have explored their connections and have proposed algorithms that use the Floyd-Warshall algorithm as a subroutine for solving the SCP, to compute the transitive closure of the incidence and distance matrix of the set that is in operation [12].

The facility plant location problem (FPLP) can be addressed with capacity constraints thus becoming a Capacitated Facility Plan Location

Generic CFLP pseudocode

```
MODEL:
MIN=@SUM(FRACTION:X*COST) +@SUM (PLANT:
Y*FIXCOST);
@FOR(CITY2(J): @SUM(CITY1(I):X (I, J))
=1);
@FOR(CITY1(I): @SUM(CITY2(J): DEMAND*X
(I, J) <=CAPACITY*Y(I));
@FOR (CITY1(I): @FOR(CITY2(J):X (I, J)
<=1));
@FOR (PLANT: @BIN(Y););
END
SETS:
CITY1/:;
CITY2/:;
FRACTION (CITY1, CITY2): COST, X;
PLANT(CITY1): Y, FIXCOST, CAPACITY;
DEMAND(CITY2), DEMAND;
ENDSETS
DATA:
COST=@file(cpl.txt)
FIXCOST=@file(cpl1.txt);
CAPACITY=@file(cpl2.txt);
DEMAND=@file(cpl3.txt);
ENDDATA
```

Fig. 1. Generic CFPL pseudocode

(CFPL) by adding a limited capacity constraint for each plant to satisfy customer demands [15,16].

3 Methodology CFPL

The CFPL consists of locating several facilities that meet the demand of certain sites at a minimum cost, each site has a certain demand, and each facility has a certain capacity.

The cost of a facility is defined as the sum of a fixed opening price and allocation prices due to transportation between the facility and each site it offers.

Next, the mathematical model for the problem is shown and the following decision variables are defined:

Let x_{ij} the decision variable represent the fraction of the annual demand of market j satisfied by plant i and a binary decision variable:

$$y_i = \begin{cases} 1 & \text{if it is open plant } i, \\ 0 & \text{other cases.} \end{cases} \quad (1)$$

Where:

c_{ij} are the transportation costs per year of shipping from each plant i to sales district j.

f_j are the fixed operating costs of each plant i

The problem can be modeled as follows:

$$\min \sum_{i \in V_1} \sum_{j \in V_2} c_{ij} x_{ij} \quad + \sum_{i \in V_1} f_i y_i. \quad (2)$$

Subject to:

$$\sum_{i \in V_1} x_{ij} = 1, \; j \in V_2, \quad (3)$$

$$\sum_{j \in V_2} d_j x_{ij} \leq q_i y_i \quad i \in V_1, \quad (4)$$

$$0 \leq x_{ij} \leq 1, \quad i \in V_1, \quad j \in V_2, \quad (5)$$

$$y_i \in \{0, 1\}, \qquad i \in V_1. \quad (6)$$

Where:

$$V_1 = V_2 \text{ the set of clients.} \quad (7)$$

The equation 2 is the objective function, and 3 is the constraint to complete the 100% of the user's demand, constrain 4 shows the satisfied demand d_j of users and q_i is the production capacity, constrain 5 shows the portion of the demand that must be sent from distribution center i to customer j. constraint 6 is the binary variable of opening a distribution center in city 1, finally, constrain 7, point out that any city in the group is a potential site to install a distribution center.

3.1 Code to CFPL

Figure 1 contains the Lingo code to solve CFPL with the branch and bound method [16]. The Lingo code is divided into 3 blocks: in the first block is the objective function and the constraints represented by @ sum and @ for, subsequently, the second block has the data structure for the lines of code, starting with sets that it contains the parameters of fixed costs, capacity, demand, the cost matrix, and the decision variables and ending with final set.

The last block starts with DATA which contains the calls to the data set with @ file and thus ends with end data.

Lingo solves the CFPL with Branch and Bound and returns an exact value in the result of the objective function and implicitly produces the location of the problem.

4 Case Study

In this work, the case that is resolved is the distribution of diverse products, using the CFPL model, whose main objective is to open distribution centers to satisfy customer demand.

Customer demand can be satisfied from various distribution centers to cover the demand of 100% of the users, therefore, a binary location variable is needed to open a new distribution center and a variable between 0 and 1 to determine the shipping percentage.

For this hypothetical case study, there are 25 cities within the State of Puebla, where $V_1 = V_2$ and V_1 is the set of clients, then V_2 is the set of potential distribution centers. Therefore, the cities set is: Ozelonacaxtla, Pahuatlán de Valle, Palmar de Bravo, Palmarito Tochapan, Pantepec, Papaloctipan, Papatlazolco, Paso Carretas, Paso Nacional, Patla, Pericotepec, Petlalcingo, Pezmatlán, Piaxtla, Pochálcatl, Primero de Mayo, Progreso 1, Progreso 2 , Progreso de Juárez, Puebla, Vista Hermosa, Quamila, Quecholac, Quechulac, Rafael J. García the aforementioned cities will be listed from 1 to 25 represented by the set code $V_1 = \{1,2,3, \dots ,25\}$

Figure 2 shows the general map of Mexico in the upper left corner and below is an extract of the State of Puebla with all its numbered cities.

The highlighted boxes with the letters A, B and C are where the locations of the cities are located to locate the distribution centers in addition to also place the clients whose demands will be satisfied according to the CFLP.

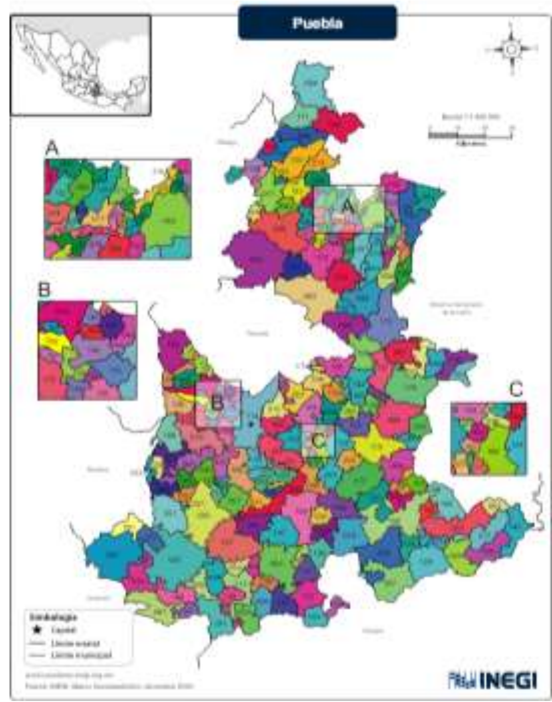

Fig. 2. Map of Puebla and division of cities [17]

5 Results and Discussion

The results obtained for the case study were the following to locate the cities to the distribution centers: of the 25 cities, the locations of Pahuatlan de Valle, Palamarito Tochapan, Paso Nacional, Patla, Pericotpec, Pezmatlan, Primero de Mayo, Progreso, Puebla, Vista Hermosa and Rafael García.

The CFPL model has the characteristic of producing a partition of the set of 25 cities, where the distribution center (DC) is located in one of the cities of each group, each DC is located in the first position of each group where the demand of each of the remaining cities is satisfied, said cities are the clients assigned to the distribution centers, therefore, we can interpret the solution of the case study as a collection of groups (G) that form a partition shown as follows in table 1.

Table 2 indicates only the distribution centers selected from each group that will supply the other cities.

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Table 1. Distribution center by group

G	*Distribution Centers by Groups*
1	*Pahuatlan, Papatlazolco,Quamila*
2	*Palmarito, Palmar, Quecholac*
3	*Paso Nacional, Paso carretas, Progreso 2*
4	*Patla, Pantepec,Papaloctipan, Progreso de Juárez, Quamila*
5	*Pericotepec, Palmar de Bravo, Paso Carretas*
6	*Pezmatlan, Primero de Mayo, Papatlazolco, Progreso de Juárez*
7	*Primero de mayo, Papatlazolco, Primero de mayo, Progreso de Juárez*
8	*Progreso 1, Petlalcingo,Piaxtla*
9	*Puebla, Papatlazcolco, Petlalcingo, Pochalcatl*
10	*Vista Hermosa, Patla, Pochalcatl*
11	*Progreso 2, Quecholac,Rafael García*

Table 2. Group of selected distribution centers

N°	*Distribution centers*
1	*Pahuatlan*
2	*Palmarito*
3	*Paso Nacional*
4	*Patla*
5	*Pericotepec*
6	*Pezmatlan*
7	*Primero de mayo*
8	*Progreso 1*
9	*Puebla*
10	*Vista Hermosa*
11	*Progreso 2*

Table 3. Scheduling shipments of demand percentages to clients

1	*Clients*									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
2	-	*100*	-	-	-	-	*27*	-	-	-
4	-	-	*34*	*100*	-	-	-	-	-	-
9	-	-	-	-	-	-	-	*48*	*100*	-
10	-	-	-	-	*100*	*100*	-	-	-	-
11	-	-	*66*	-	-	-	-	*52*	-	-
13	*100*	-	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	*55*	-	-	-
17	-	-	-	-	-	-	-	-	-	-
20	-	-	-	-	-	-	*18*	-	-	-
21	-	-	-	-	-	-	-	-	-	*100*
25	-	-	-	-	-	-	-	-	-	-

Table 3 is presented below with the data of the aforementioned groups, in addition to the scheduling of shipments from the distribution centers to the customers' cities to satisfy demand. A sample was taken from the customers' 11 distribution centers (DC) and 10 cities. The total cost of the objective function for the entire operation of the problem is $939,136.5, expressed in Mexican national currency, the result was obtained by the Branch and bound method with a total of 4,459 iterations processed in Lingo.

The same code was executed for other instances with the branch and bound method using the same demand but changing the supply

Table 4. Instances with same demanda but diferente capacity

N°	Capacity	Obj.V.	DC	Itera.
1	*605982*	*939137*	*11*	*4459*
2	*484786*	*1199970*	*14*	*6672*
3	*454487*	*1289860*	*15*	*4736*

Table 5. MILP GOSF Model

Model MILP GOSF			
n	OV	OB	Infe
26	91942.6	91942.6	0
28	1002278	1002278	0
29	1017704	1017704	0
31	1093483	1093483	0
33	1177234	1177234	0
34	1189016	1189016	0
35	1256674	1256674	0

Table 6. MILP GOSF Model

Model MILP GOSF						
n	ESS	TSI	ERS	TV	IV	TC
26	565	12293	1.93	702	26	729
28	6932	131822	8.95	812	28	841
29	15	2799	0.48	870	29	900
31	2934	54090	5.2	992	31	1024
33	49178	870277	52.9	1122	33	1156
34	41	2878	0.86	1190	34	1225
35	2068009	53096224	4857.62	1260	35	1296

capacity to see the behavior of the changes as shown in table 4.

The table shows the three instances with different capacities and marks for each instance its objective values and iterations, as well as the distribution centers (DC). According to the table, the first instance points to a capacity of 605982 with a target value of 939137, locating the distribution center in city11 with 4459 iterations, the following data of the other instances can be interpreted in the same way.

Table 5 and 6 shows another technique applied to a similar case with data applied to the mixed integer lineal program (MILP) model for a set of seven instances of greater dimension to the case study. As can be seen in column 4 table 5, no infeasible solutions are presented and, in all cases, global optimal solutions are achieved, thus demonstrating the robustness of the model solutions.

Within table 5 it can be seen the title represented by the MILP model and the global optimal solution found (GOSF) the columns represent the following: n the dimension of the instances of the case, OV is the objective function, OB represents the objective bound which for this case is equal to the objective function, Infe represents the infeasibilities.

Within table 6 shows: ESS is the extended solver steps, followed by the total solver iterations TSI, then there is elapsed runtime seconds ERS, Total variables TV, as well as integer variable IV, finally the total constraints TC.

It is worth mentioning that organizations frequently seek to maximize their benefits and minimize costs in their different areas. The application of these case studies allows us to exemplify the logistics operations within companies and help them direct their efforts towards efficient decision-making aligned with achieving their organizational goals. Therefore, in the logistics area there are three types of decisions: strategic, tactical and operational, so the presented model impacts strategic decisions, since the logistics system will prevail in long-term decision-making.

Additionally, the model impacts cost minimization in the transfer of users to distribution centers, therefore having an impact on the economic scope of the operation.

6 Conclusions and Future Work

This work addresses the CFPL modeled as an integer programming formulation that is solved by means of branch and bound method [8] with Lingo. A hypothetical case is presented using 25 cities as a test for the distribution of products, the central point is to locate new distribution centers that satisfy the demand of customers located in the same cities.

Within logistics problems there are three levels of decision, among them are operational, tactical and strategic, for this particular case strategic decisions were used since locating distribution centers are decisions that do not occur so frequently.

The main objective of this application focused on locating distribution centers within 25 cities that, in turn, would be considered as clients to satisfy their demands. In such a way that a mixed integer programming model of the location - allocation CFPL assignment type and locate the new distribution centers was used as a tool for strategic decision making.

The results of the execution of the Lingo sequential code determined the efficient structure of the distribution system, assigning the distribution centers to the entities and their respective associates, in such a way that distribution costs were minimized throughout the system, as shown in section 5.

As future work, it is planned to design randomly generated test instances, but in particular to generate customer demand by forecasting methods.

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