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Methodology for Measuring the Real-Time Transfer Function of Dynamic Systems in Operation

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Abstract. This paper presents a robust methodology to measure the transfer function of a dynamic system in operation by applying a delta signal as input. The impulse response of the systems is analyzed in the Laplace domain, thus allowing us to obtain key information about their behavior, stability and performance. A comparative approach is proposed between the nominal transfer function and its measurement to detect faults or deficiencies, evaluating metrics such as gain margin and phase. In addition, a frequency domain analysis is used to identify alterations in the system dynamics. This methodology is useful for diagnosing and maintaining control systems, improving their reliability and robustness against operational variations and structural failures of the system.

Keywords. Transfer function, dynamic systems, delta signal, impulse response, frequency analysis, system stability, fault diagnosis, robust control, system identification, disturbances.

1 Introduction

In the field of control engineering and computer systems theory, the transfer function is one of the primary tools for modeling and analyzing the dynamic behavior of linear time-invariant systems. This mathematical representation relates the input and output of a system in the frequency domain, allowing engineers to predict the system's response to various stimuli and thus design efficient control strategies [1].

However, one of the main challenges in practice is the presence of various external or internal disturbances, which can significantly alter the behavior of the system and, consequently, the accuracy of the model based on the transfer function [2].

Disturbances, whether environmental, instrumental, or inherent to the process, introduce uncertainty into the system, which can lead to various discrepancies between the theoretical model and the system's actual behavior.

This phenomenon is particularly critical in applications requiring high precision due to their high-risk nature, such as aircraft control, energy systems, or complex industrial processes, where even small deviations can have significant consequences [3]. Understanding how disturbances affect the transfer function and, consequently, the system model is a problem of great theoretical and practical relevance [4].

Various research articles have proposed different methodologies to address this challenge, ranging from robustness and adaptive control techniques to approaches based on identifying systems in the presence of noise. For example, some articles analyze the impact of stochastic disturbances on the stability of linear systems [6], while others propose a method to compensate for disturbances in nonlinear systems using significantly more robust controllers [7].

Our article seeks to contribute to bridging this gap through a systematic analysis of the effects of disturbances on the transfer function and their influence on the accuracy and reliability of the model.

The main objective of this work is to investigate how the introduction of disturbances into a system alters its transfer function and, consequently, its dynamic model.

To this end, a theoretical framework is proposed that allows us to quantify these changes, and case studies are presented that illustrate the magnitude of the deviations in various scenarios. The results obtained not only enrich our theoretical understanding of the problem but also provide more practical tools for the design of more robust and resilient control systems.

2 Theoretical Framework

The transfer function is one of the most fundamental mathematical tools in control theory and dynamic systems. It represents the relationship between the input and output of a linear time-invariant (LTI) system in the frequency domain. For a continuous system, the transfer function G(s) is defined as the ratio of the Laplace transform of the output Y(s) to the Laplace transform of the input U(s), under the assumption of zero initial conditions [1]:

$$G(s) = \frac{Y(s)}{U(s)}. \tag{1}$$

In Eq. 1, we can observe that the representation is particularly useful because it allows us to analyze key system properties, such as stability, transient response, and frequency response, without needing to solve time-domain differential equations [2]. Furthermore, the transfer function facilitates controller design using techniques such as root locus analysis and Bode plotting [3].

However, the transfer function has some inherent limitations. In particular, it assumes that the system is linear, time-invariant, and not subject to external or internal disturbances. These assumptions are rarely met in practical applications, leading to a discrepancy between the theoretical model and the actual behavior of the system [4].

Disturbances are unwanted signals that primarily affect the behavior of a dynamic system. They can be classified into two main categories: deterministic disturbances and stochastic disturbances. Deterministic disturbances are predictable signals that can be mathematically modeled; some common examples include step, ramp, and sinusoidal signals. These disturbances are usually associated with abrupt changes in operating conditions, such as variations in the load of a motor or fluctuations in the temperature of an industrial process [5].

Stochastic disturbances are random signals that cannot be predicted with certainty; for example, white noise, thermal noise, and random vibrations. These disturbances are particularly challenging because they require advanced modeling and control techniques, such as Kalman filtering or robust control [6].

Disturbances can affect the system in various ways. First, they can alter the effective transfer function of our system, leading to unpredictable behavior. Second, they can excite unmodeled modes of the system, which can result in instability or performance degradation [7]. Finally, disturbances can introduce errors in the measurement of the system output, complicating the design of feedback-based control strategies [8].

One of the critical properties in the design of control systems is robustness. This refers to a system's ability to maintain its performance in the presence of various uncertainties and disturbances. Within the context of the transfer function, robustness is evaluated using metrics such as gain margin and phase margin, which indicate how much the system can vary before becoming unstable [9].

Gain margin is the factor by which the system's gain can be multiplied before it becomes unstable. A high gain margin indicates that the system can tolerate large variations in gain without losing stability. Phase margin, on the other hand, is the additional phase angle that can be added to the system before it becomes unstable.

A high phase margin indicates that the system can tolerate delays or phase shifts without losing stability. In addition to these classic metrics, advanced techniques have been developed to evaluate robustness, such as singular value analysis and H_∞ synthesis.

These techniques allow us to design controllers that guarantee acceptable performance even in the presence of disturbances and uncertainties [10].

When systems are subject to disturbances, their effective transfer function can be significantly altered. For example, additive disturbances at the system input modify the input-output relationship, resulting in an apparent transfer function G'(s) that differs from the nominal G(s). This change can lead to errors in predicting system behavior and, in some extreme cases, to system instability [11].

Furthermore, disturbances can affect the poles and zeros of the transfer function, thus modifying the system's frequency response. This is particularly critical in feedback control systems, where small changes in the transfer function can be amplified and lead to oscillations or divergences [12].

To address the problem of disturbances in control systems, various techniques have been developed, among which those listed in Table 1 stand out:

Table 1. Disturbance Mitigation Techniques

Mitigation Techniques	Description
Adaptive Control	This technique adjusts the controller parameters in real time to compensate for disturbances and variations in the system.
Disturbance Filtering	Uses filters (e.g., the Kalman filter) to estimate and eliminate disturbances from the control signal.
Robust Control	Controllers are designed to guarantee acceptable performance even in the presence of disturbances and uncertainties.
System Identification	Uses experimental data to estimate the system's transfer function in the presence of disturbances. This technique is useful when the theoretical model of the system is inaccurate.

These techniques have proven effective in a wide range of applications, from flight control systems to complex industrial processes.

3 Methodology

Measuring the transfer function of a dynamic system in operation is done by applying a Dirac delta signal δ(t) to the system input. The delta δ(t) has properties that make it ideal for analyzing a system's response. In its frequency domain, the Laplace transform of a δ(t) is equal to 1, which, in addition to simplifying the analysis of linear time-invariant (LTI) systems, allows us to subject a system to an infinite number of test signals simultaneously.

As shown in Equation 1, a system in the Laplace domain is a system represented by its nominal transfer function, where:

- G(s) is the system's transfer function,
- Y(s) is the Laplace transform of the output y(t),
- U(s) is the Laplace transform of the input u(t).

Since applying the delta signal δ(t) to the system results in U(s) = 1, we can directly measure the system's impulse response. Therefore, the system's output in the Laplace domain will be:

$$Y(s) = G(s) \cdot U(s) = G(s). \tag{2}$$

Applying the inverse Laplace transform to Y(s) yields the system's impulse response in the time domain, y(t). This provides key information about the system's behavior:

- Poles and zeros, which determine the system's stability and dynamics,
- Settling time or steady state,
- Transient response,

$$y(t) = \mathcal{L}^{-1}[G(s)]. \tag{3}$$

To evaluate system failures or deficiencies, a comparative analysis is performed between the nominal and measured transfer functions. The deviation between these two functions can be modeled as in equation 4.

$$\Delta G(s) = G_{real}(s) - G_{nominal}(s). \quad (4)$$

Where it indicates the alterations caused by disturbances, failures, or degradations in the system. If we obtain a large magnitude in equation 4, it may be indicative of significant failures.

$$|G_{real}(jw)| - |G_{nominal}(jw)|, \quad \angle G_{real}(jw) - \angle G_{nominal}(jw). \quad (5)$$

Equation 5 shows the differences that can reveal frequencies where the system has certain resonance problems, insufficient damping, or degradation.

Performance metrics allow us to evaluate whether the system meets the expected requirements in terms of stability, robustness, and responsiveness. These metrics are derived from the analysis of the transfer function and its impulse or frequency response.

The gain margin measures the system's robustness against gain increases before the system becomes unstable. We evaluate it at the phase crossover frequency (ω_ϕ), defined as the frequency where the phase of our transfer function reaches -180°:

$$G_m = \frac{1}{|G(j\omega_\phi)|}. \quad (6)$$

Interpreting equation 6 is as follows.

If $G_m > 1$, it tells us that the system is robust to gain increases.

If $G_m \approx 1$, we interpret that the system is on the verge of instability.

If $G_m < 1$, it means that the system is unstable.

Our phase margin indicates the amount of phase gain that can be lost before the system becomes unstable. We measure the gain crossover frequency (ω_g), which is the frequency where |G(jω)| = 1:

$$\phi_m = 180^{\circ} + \angle G(j\omega_g). \quad (7)$$

Interpreting equation 7, we observe that:

A $\phi_m > 30°$ ensures good damping and stability.

If $\phi_m \approx 0°$, our system is close to being unstable.

While $\phi_m < 0°$ indicates instability.

The passband can be defined as the range of frequencies where the system responds adequately (for example, when the gain remains above a level defined as -3 dB relative to its maximum gain):

$$BW = \left\{\omega : |G(j\omega)| \geq \frac{1}{\sqrt{2}} \cdot |G(0)|\right\}. \quad (8)$$

Interpreting equation 8, we understand that a wider passband implies the system can handle higher frequency signals without significant attenuation. Conversely, a very narrow passband may indicate deficiencies in response speed.

Our settling time measures how long it takes for the system's response to remain within a specific range (2% or 5%) of the final value after a disturbance.

For second-order systems, we can make the following estimate, as shown in equation 9:

$$T_s \approx \frac{4}{\zeta \omega_n}. \quad (9)$$

Where:

ζ is the damping coefficient and ω_n is the natural frequency of the system.

Applied to real systems, external (d(y)) or internal (n(t)) disturbances significantly affect system performance. In a more detailed analysis, these disturbances are modeled as additional signals in the system:

$$Y(s) = G(s)U(s) + D(s) \quad (10)$$

The system with disturbances can be represented as shown in Equation 10, where:

- Y(s) is the total output,
- G(s) is the system's transfer function,
- U(s) is the control input,
- D(s) is the Laplace transform of the disturbance.

If the disturbances affect the output, the system's effective transfer function is modified.

To isolate the effect of the disturbances, linear superposition is used:

$$Y(s) = Y_{control}(s) + Y_{perturbación}(s) \quad (11)$$

Where we interpret equation 11, in the following way:

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– $Y_{control}(s) = G(s)U(s)$,
– $Y_{perturbación}(s) = D(s)$.

The disturbance can be identified and filtered using adaptive control techniques.

For the frequency domain, we analyze disturbances based on their spectrum. The disturbances D(s) can be modeled as white noise or low-frequency (1/f) sinusoidal signals:

$$|D(j\omega)| \approx \frac{A}{\omega^{\beta}}. \quad (12)$$

In equation 12, our variable β depends on the type of noise:

β=0: White noise.

β=1: Pink noise.

β=2: Brown noise.

To compensate for disturbances, it is possible to design a controller that minimizes the influence of D(s). If our system includes a C(s) type controller, the closed model would be as shown in equation 13:

$$T(s)=(G(s)C(s))/(1+G(s)C(s)). \quad (13)$$

The main design objective is for T(s) to reduce the system's sensitivity to D(s). This implies high gain at low frequencies to reject persistent disturbances, and low gain at high frequencies to avoid noise amplification.

Applying these suggested metrics and models provides a solid theoretical tool to diagnose failures, compensate for disturbances, and ensure adequate system performance under adverse real-world conditions.

4 Conclusion

With this proposed methodology, we have established a comprehensive approach for performing real-time measurements based on the transfer function of dynamic systems in operation. By using delta signals and frequency response analysis, we achieve a more accurate representation of a system's behavior, even under the influence of external disturbances and uncertainties. This capability allows us not only to identify faults and deficiencies in systems, but also to develop robust and resilient control strategies.

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