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Fractional Order PID Control Law for Trajectory Tracking Using Fractional Order Time-Delay Recurrent Neural Networks for Fractional Order Complex Dynamical Systems

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Abstract. This paper presents an extension to the theory on the analysis of stability of complex systems with time delay of fractional order, the previous study is based on the theory of functions of Lyapunov-Krasovski and the Fractional Order PID control law. The analytical results are illustrated by the simulation of fractional nonlinear systems interconnected with time delay which are forced to follow complex trajectories as chaotic systems.

Fractional complex dynamical systems, trajectory tracking, Fractional Order Lyapunov-Krasovskii theory, Fractional Order PID control law.

1 Introduction

This paper analyzes the path not for a non-linear system but for a network of nonlinear systems coupled with delay, which are forced to follow a reference signal generated by a non-linear chaotic system. The control law that guarantees trajectory tracking is obtained using the Lyapunoc-Krasovskii methodology and the PID control law. The control law that guarantees trajectory tracking is obtained by using the Lyapunov methodology and the Fractional Order PID Control Law. It is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers. The proportional action tends to stabilize the system, while the integral control action tends to eliminate or reduce steady-state error in response to various inputs.

Derivative control action, when added to a proportional controller, provides a means of obtaining a controller with high sensitivity. An advantage of using derivative control action is that it responds to the rate of change of the actuating error and can produce a significant correction before the magnitude of the actuating error becomes too large.

Derivative control thus anticipates the actuating error, initiates an early corrective action, and tends to increase the stability of the system. The combination of proportional control action, integral control action, and derivative control action is termed proportional-plus-integral-plus-derivative control action. It has the advantages of each of the three individual control actions.

A Fractional Order PID controller, also known as a $\left[PI^{\lambda}D^{\alpha}\right]$ controller, takes on the form [1]:

$$u(t) = K_p e(t) + K_i a D_t^{-\lambda} e(t) + K_d a D_t^{\alpha} e(t),$$

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where λ and α are the fractional orders of the controller and $e(t)$ is the system error. Note that the system error $e(t)$ replaces the general function $f(t)$.

The analysis and control of complex behavior in complex networks, which consist of dynamical nodes, has become a point of great interest in recent studies [2, 3, 4]. The complexity in networks comes from their structure and dynamics but also from their topology, which often affects their function.

Recurrent neural networks have been widely used in the fields of optimization, pattern recognition, signal processing and control systems, among others. They have to be designed in such a way that there is one equilibrium point that is globally asymptotically stable.

Trajectory tracking is a very interesting problem in the field of theory of systems control; it allows the implementation of important tasks for automatic control such as: high speed target recognition and tracking, real-time visual inspection, and recognition of context sensitive and moving scenes, among others. We present the results of the design of a control law that guarantees the tracking of general fractional order complex dynamical networks.

2 Mathematical Models

2.1 Fractional General Complex Dynamical Network

In this work we use Caputo's fractional operator which is defined, for 0 $<\alpha<$1, by:

$$x^{(\alpha)}(t) = {}_0^c D_t^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t x^{'}(\tau)(t-\tau)^{-\alpha}d\tau.$$

If $x(t) \in \mathbb{R}^n$, we consider that $x^{(\alpha)}(t)$ is the Caputo fractional operator applied to each entry:

$$x^{(\alpha)}(t) = ({}_0^c D_t^\alpha x_{i1}(t), ..., {}_0^c D_t^\alpha x_{in}(t))^T.$$

Consider a network consisting of N linearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system, described by:

$$x_i^{(\alpha)} = f_i(x_i) + \sum_{\substack{j=1 \\ j\neq i}}^N c_{ij} a_{ij} \Gamma(x_j - x_i), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in \mathbb{R}^n$ are the state vectors of node i, $f_i : \mathbb{R}^n \longmapsto \mathbb{R}^n$ represents the self-dynamics of node i, constants $c_{ij} > 0$ are the coupling strengths between node i and node j, with $i, j = 1, 2, \ldots, N$.

$\Gamma = (\tau_{ij}) \in \mathbb{R}^{n\times n}$ is a constant internal matrix that describes the way of linking the components in each pair of connected node vectors $(x_j - x_i)$: that is to say for some pairs (i, j) with $1 \leq i, j \leq n$ and $\tau_{ij} \neq 0$ the two coupled nodes are linked through their ith and jth sub-state variables, respectively.

While the coupling matrix $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ denotes the coupling configuration of the entire network: that is to say if there is a connection between node i and node $j (i \neq j)$, then $a_{ij} = a_{ji} = 1$; otherwise $a_{ij} = a_{ji} = 0$.

2.2 Fractional Time-Delay Recurrent Neural Network

Consider a fractional delayed recurrent neural network in the following form:

$$\begin{aligned} x_{n_i}^{(\alpha)} &= A_{n_i} x_{n_i} + W_{n_i} \sigma(x_{in}(t-\tau) + u_{in} + \\ &\quad \sum_{\substack{j=1 \\ j\neq i}}^N c_{injn} a_{injn} \Gamma(x_{jn} - x_{in}), \\ i &= 1, 2, \ldots, N, \end{aligned} \tag{2}$$

where $x_{in} = (x_{in_1}, x_{in_2}, \ldots, x_{in_n})^T \in \mathbb{R}^n$ is the state vector of neural network i, $u_{in} \in \mathbb{R}^n$ is the input of neural network i, $A_{in} = -\lambda_{in} I_{n\times n}$, $i = 1, 2, \ldots, N$, is the state feedback matrix, with λ_{in} being a positive constant, $W_{in} \in \mathbb{R}^{n\times n}$ is the connection weight matrix with $i = 1, 2, \ldots, N$, and $\sigma(\cdot) \in \mathbb{R}^n$ is a Lipschitz sigmoid vector function [5, 6], such that $\sigma(x_{in}) = 0$ only at $x_{in} = 0$, with Lipschitz constant L_{σ_i}, $i = 1, 2, \ldots, N$ and neuron activation functions $\sigma_i(\cdot) = \tanh(\cdot)$, $i = 1, 2, \ldots, N$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in \mathbb{R}^n$ are the state vectors of node i, $f_i : \mathbb{R}^n \longmapsto \mathbb{R}^n$ represents the self-dynamics of node i, constants $c_{ij} > 0$ are the coupling strengths between node i and node j, with $i, j = 1, 2, \dots, N$. $\Gamma = (\tau_{ij}) \in \mathbb{R}^{n \times n}$ is a constant internal matrix that describes the way of linking the components in each pair of connected node vectors $(x_j - x_i)$: that is to say for some pairs (i, j) with $1 \leq i, j \leq n$ and $\tau_{ij} \neq 0$ the two coupled nodes are linked through their ith and jth sub-state variables, respectively.

While the coupling matrix $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ denotes the coupling configuration of the entire network: that is to say if there is a connection between node i and node $j (i \neq j)$, then $a_{ij} = a_{ji} = 1$; otherwise $a_{ij} = a_{ji} = 0$.

3 Trajectory Tracking

The objective is to develop a control law such that the ith fractional delayed neural network (2) tracks the trajectory of the ith fractional dynamical system (1). We define the tracking error as $e_i = x_{in} - x_i$, $i = 1, 2, \dots, N$ whose time derivative is:

$$e_i^{(\alpha)} = x_{in_i}^{(\alpha)} - x_i^{(\alpha)}, \quad i = 1, 2, \dots, N. \tag{3}$$

From (1, 2, 3), we obtain:

$$\begin{aligned} e_i^{(\alpha)} \quad = \quad & A_{in} x_{in} + W_{in} \sigma(x_{in}(t - \tau)) + u_{in} - f_i(x_i) + \\ & \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{injn} a_{injn} \Gamma(x_{jn} - x_{in}) - \\ & \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} a_{ij} \Gamma(x_j - x_i), \quad i = 1, 2, \dots, N. \end{aligned} \tag{4}$$

Adding and subtracting, $W_{in} \sigma(x_i(t - \tau))$, $\alpha_i(t)$, $i = 1, 2, \dots, N$, to (4), where α_i is defined below, and considering that $x_{in} = e_i + x_i$, $i = 1, 2, \dots, N$, then:

$$\begin{aligned} e_i^{(\alpha)} = & A_{in} e_i + W_{in}(\sigma(e_i + x_i(t - \tau)) - \sigma(x_i(t - \tau))) + \\ & (u_{in} - \alpha_i) + (A_{in} x_i + W_{in} \sigma(x_i(t - \tau)) + \alpha_i) - \\ & f_i(x_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{injn} a_{injn} \Gamma(x_{jn} - x_{in}), \end{aligned} \tag{5}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} a_{ij} \Gamma(x_j - x_i), \quad i = 1, 2, \dots, N.$$

In order to guarantee that the ith neural network (2) tracks the ith reference trajectory (1), the following assumption has to be satisfed:

Assumption 1. There exist functions $\rho_i(t)$ and $\alpha_i(t)$, $i = 1, 2, \dots, N$, such that:

$$\begin{aligned} \rho_i^{(\alpha)}(t) &= A_{in} \rho_i(t) + W_{in} \sigma(\rho_i(t)) + \alpha_i(t) \\ \rho_i(t) &= x_i(t), \quad i = 1, 2, \dots, N. \end{aligned} \tag{6}$$

Let's us define:

$$\begin{aligned} \widetilde{u}_{in} &= (u_{in} - \alpha_i) \\ \phi_\sigma(e_i, x_i(t - \tau)) &= \sigma(e_i + x_i(t - \tau)) - \sigma(x_i(t - \tau)), \\ i &= 1, 2, \dots, N. \end{aligned} \tag{7}$$

From (6, 7), equation (5) is reduced to:

$$\begin{aligned} e_i^{(\alpha)} = \quad & A_{in} e_i + W_{in} \phi_\sigma(e_i, x_i(t - \tau)) + \widetilde{u}_{in} + \\ & \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{injn} a_{injn} \Gamma(x_{jn} - x_{in}) - \\ & \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} a_{ij} \Gamma(x_j - x_i), \end{aligned} \tag{8}$$

$$i = 1, 2, ..., N.$$

We can also write:

$$\begin{aligned} & \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{injn} a_{injn} \Gamma(x_{jn} - x_{in}) \\ = \quad & \Gamma(\sum_{\substack{j=1 \\ j \neq i}}^{N} c_{injn_j} a_{injn} x_{jn} - x_{in} \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{injn} a_{injn}) \\ & \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} a_{ij} \Gamma(x_j - x_i), \\ = \quad & \Gamma(\sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} a_{ij} x_j - x_i \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} a_{ij}), \\ i \quad = \quad & 1, 2, \dots, N, \end{aligned} \tag{9}$$

where we used that $c_{injn} = c_{ij}$ and $a_{injn} = a_{ij}$. Then, with the above equation, equation (8) becomes:

$$\begin{aligned} e_i^{(\alpha)} &= A_{in}e_i + W_{in}\phi_\sigma(e_i, x_i(t-\tau)) + \widetilde{u}_{in} + \\ &\quad \Gamma(\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_j - e_i\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}), \qquad (10) \\ &= A_{n_i}e_i + W_{in}\phi_\sigma(e_i, x_i(t-\tau)) + \widetilde{u}_{in} + \\ &\quad \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\Gamma(e_j - e_i), \\ i &= 1, 2, \ldots, N. \end{aligned}$$

It is clear that $e_i = 0$, $i = 1, 2, \ldots, N$ is an equilibrium point of (10), when $\widetilde{u}_{in} = 0$, $i = 1, 2, \ldots, N$. Therefore, the tracking problem can be restated as a global asymptotic stabilization problem for the system (10).

4 Tracking Error Stabilization and Control Design

In order to establish the convergence of (10) to $e_i = 0$, $i = 1, 2, \ldots, N$, which ensures the desired tracking, first, we propose the following candidate Lyapunov-Krasovskii function [7]:

$$\begin{aligned} V_N(e) &= \sum_{i=1}^{N} V(e_i) = \\ &\quad \sum_{i=1}^{N} \tfrac{1}{2}[(e_i^T, w_i^T)(e_i, w_i)^T] + \qquad (11) \\ &\quad \int_{t-\tau}^{t} (\phi_\sigma^T(s)W_{n_i}^T W_{n_i}\phi_\sigma(s))ds. \end{aligned}$$

In fractional calculus, the product rule for the derivative is no longer valid. However, we still have an upper bound for the product that appears in (11). Specifically, from Lemma 1 in [8] the time derivative of (11), along the trajectories of (10), and adding the Derivative "D":

$$\begin{aligned} aD_t^\alpha V =& e_i^T aD_t^\alpha e_i + w_i^T aD_t^\alpha w_i + \\ & \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) - \\ & \phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau), \end{aligned}$$

$$\begin{aligned} aD_t^\alpha V =& e^T[aD_t^\alpha e_i + K_d aD_t^\alpha e_i(t)] + w_i^T aD_t^\alpha w_i + \\ & \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) - \\ & \phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau), \end{aligned}$$

$$\begin{aligned} aD_t^\alpha V =& e_i^T[1 + K_d]aD_t^\alpha e_i(t) + w_i^T aD_t^\alpha w_i + \\ & \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) - \qquad (12) \\ & \phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau), \end{aligned}$$

If $a = [1 + K_d], \alpha = \lambda$, and $w_i = K_i aD_t^{-\alpha} e_i(t)$, then $aD_t^\alpha w_i = K_i e(t)$, [9]

$$\begin{aligned} aD_t^\alpha V =& \sum_{j=1}^{N} ae_i^T(A_{in}e_i + W_{in}\phi_\sigma(e_i, x_i(t-\tau)) + \\ & \widetilde{u}_{in} + \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}\Gamma(e_j - e_j)) + w_i^T K_i e(t) + \\ & \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) - \\ & \phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau). \end{aligned}$$

We can then write:

$$\begin{aligned} aD_t^\alpha V =& \sum_{i=1}^{N}\Big(-a\lambda_{i_n}\|e_i\|^2 + \\ & æ_i^\intercal W_{i_n}\phi_\sigma(e_i, x_i) + ae_i^\intercal\widetilde{u}_{i_n} + \\ & a\left(\sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}a_{ij}e_i^\intercal\Gamma(e_j - e_j)\right) + w_i^T K_i e(t) \\ & \phi_\sigma^T(t)W_{n_i}^T W_{n_i}\phi_\sigma(t) - \\ & \phi_\sigma^T(t-\tau)W_{n_i}^T W_{n_i}\phi_\sigma(t-\tau). \qquad (13) \end{aligned}$$

Next, let's consider the following inequality, proved in [10, 11]:

$$X^\top Y + Y^\top X \leq X^\top \Lambda X + Y^\top \Lambda^{-1} Y, \qquad (14)$$

which holds for all matrices $X, Y \in \mathbb{R}^{n\times k}$ and $\Lambda \in \mathbb{R}^{n\times n}$ with $\Lambda = \Lambda^\top > 0$. Applying (14) with $\Lambda =$

$I_{n\times n}$ to the term $e_i^{\intercal} W_{i_n}\phi_\sigma(e_i,x_i)$, $i = 1,2,...,N$, we get:

$$\begin{aligned}&e_i^{\intercal} W_{i_n}\phi_\sigma(e_i,x_i(t-\tau)) \le \frac{1}{2}e_i^{\intercal} e_i+\\&\frac{1}{2}\phi_\sigma^\top(e_i,x_i(t-\tau))W_{i_n}^\top W_{i_n}\phi_\sigma(e_i,x_i(t-\tau))\\=&\frac{1}{2}\|e_i\|^2+\frac{1}{2}\phi_\sigma^\top(e_i,x_i(t-\tau))\times\\&W_{i_n}^\top W_{i_n}\phi_\sigma(e_i,x_i(t-\tau)),\\&i=1,2,\ldots,N.\end{aligned} \tag{15}$$

Since ϕ_σ is Lipschitz, then:

$$\|\phi_\sigma(e_i,x_i)\| \le L_{\phi_{\sigma 1}} \parallel e_i \parallel, i=1,2,\ldots,N, \tag{16}$$

with Lipschitz constant $L_{\phi_{\sigma_i}}$. Applying (16) to $\frac{1}{2}\phi_\sigma^\top(e_i,x_i)W_{i_n}^\top W_{i_n}\phi_\sigma(e_i,x_i)$ we obtain:

$$\begin{aligned}&\frac{1}{2}\phi_\sigma^\top(e_i,x_i)W_{i_n}^\top W_{i_n}\phi_\sigma(e_i,x_i),\\ \le\ & \frac{1}{2}\left\|\phi_\sigma^\top(e_i,x_i)W_{i_n}^\top W_{i_n}\phi_\sigma(e_i,x_i)\right\|,\\ \le\ & \frac{1}{2}\left(L_{\phi_{\sigma_i}}\right)^2\|W_{i_n}\|^2\|e_i\|^2,\ i=1,2,...,N.\end{aligned} \tag{17}$$

Next, (15) is reduced to:

$$\begin{aligned}&e_i^{\intercal} W_{i_n}\phi_\sigma(e_i,x_i)\\ \le&\frac{1}{2}\|e_i\|^2+\frac{1}{2}\left(L_{\phi_{\sigma_i}}\right)^2\|W_{i_n}\|^2\|e_i\|^2\\=&\frac{1}{2}\left(1+L_{\phi_{\sigma_i}}^2\|W_{i_n}\|^2\right)\|e_i\|^2,\, i=1,2,...,N.\end{aligned} \tag{18}$$

Then, we have that:

$$\begin{aligned}V_N^{(\alpha)}(e) \le \sum_{i=1}^N e^T(-a\lambda_{i_n}e_i - a\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}\Gamma e_i+\\ \frac{a}{2}\left(1+L_{\phi_{\sigma_i}}^2\|W_{i_n}\|^2\right)e_i+\\ w_i^T K_i e(t) + a\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}e_i^{\intercal}\Gamma e_j + ae_i^T\widetilde{u}_{n_i}\end{aligned} \tag{19}$$

We define $\widetilde{u}_{n_i} = \overset{\approx}{u}_i + \overset{\approx}{u}_{i_j} + K_{p_i}e_i + w_i - \frac{\gamma}{2}\left(1+L_{\phi_{\sigma_i}}^2\|W_{i_n}\|^2\right)$, $i=1,2,...,N$, and from (19) we get:

$$\begin{aligned}V_N^{(\alpha)}(e) \quad\le\quad & \sum_{i=1}^N[-a(\ \lambda_{n_i}-K_p)e_i^Te_i+\\ & \frac{a(\gamma-1)}{2}\left(1+L_{\phi_{\sigma_i}}^2\|W_{i_n}\|^2\right)e_i^Te_i+\\ & (a+K_i)e^Tw_i - a\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}\Gamma e_i^{\intercal}e_i\\ & +a\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}\Gamma e_i^{\intercal}e_j\ +ae^T\overset{\approx}{u}_i+ae^T\overset{\approx}{u}_{i_j}\qquad .\end{aligned} \tag{20}$$

Here we select $(a+K_i)=0$, so, $K_d=-K_i-1; K_d \ge 0$,then $K_i \ge -1$. With this selection of parameters (20) is reduced to:

$$\begin{aligned}aD_t^\alpha V \quad=\quad & V_N^{(\alpha)}(e) \le \sum_{i=1}^N[-a(\ \lambda_{n_i}-K_p)e_i^Te_i+\\ & \frac{a(\gamma-1)}{2}\left(1+L_{\phi_{\sigma_i}}^2\|W_{i_n}\|^2\right)e_i^Te_i-\\ & a\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}\Gamma e_i^{\intercal}e_i+\\ & a\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}\Gamma e_i^{\intercal}e_j\ +ae^T\overset{\approx}{u}_i+ae^T\overset{\approx}{u}_{i_j}\qquad .\end{aligned}$$

In this part, if $\lambda_{n_i}-K_p>0$, $a>0$, then $aD_t^\alpha V < 0, \forall e_i, w_i, W_{n_i}$,the traking error is asymptotically stable and it converges to zero for every $e_i \neq 0$; i.e. the Neural Network will follow the plant asymptotically.

Now, we propose to use the following control law:

$$\begin{aligned}\widetilde{u}_{n_i} \quad&=\quad -\left(1+L_{\phi_{\sigma_i}}^2\|W_{i_n}\|^2\right)e\\ &\quad -\sum_{\substack{j=1\\j\neq i}}^N c_{ij}a_{ij}\Gamma e_j,\\ i \quad&=\quad 1,2,...,N.\end{aligned} \tag{21}$$

· In this case, $V_N^{(\alpha)}(e) < 0$, $\forall\ e \neq 0$. This means that the proposed control law (21) can globally and asymptotically stabilize the ith error system (10), therefore ensuring the tracking of (1 by 2).

Finally, the control action of the recurrent neural networks is given by:

$$\begin{aligned} u_{i_n} &= f_i(x_i) + \lambda_{n_i} x_i - W_{n_i}\sigma\left(x_i(t-\tau)\right) + \\ &\quad \frac{1}{2}\left(1 + L^2_{\phi_{\sigma_i}}\left\|W_{i_n}\right\|^2\right) e_i + \\ &\quad K_p e(t) + K_i a D_t^{-\lambda} e(t) + K_d a D_t^{\alpha} e(t) - \\ &\quad \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij} a_{ij} \Gamma e_j, \\ i &= 1, 2, ..., N. \end{aligned} \tag{22}$$

5 Simulations

In order to illustrate the applicability of the discussed results, we consider a fractional order dynamical network with just one fractional order Lorenz's node and three identical fractional order Chen's nodes. The single fractional order Lorenz system is described by:

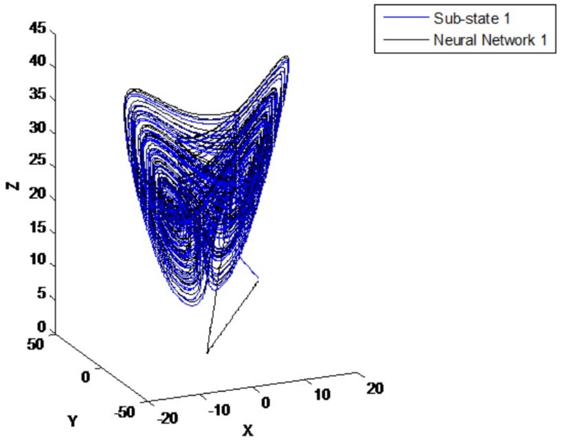

Fig. 1. Sub-State of Lorenz's attractor with initial condition X1 (0) = (10; 0; 10)ˆT

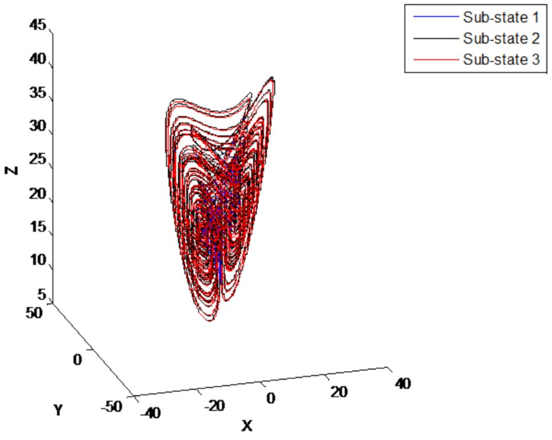

Fig. 2. Sub-States of Chen's attractor with initial condition X2,3,4(0) = (-10; 0; 37)$^{\wedge}T$

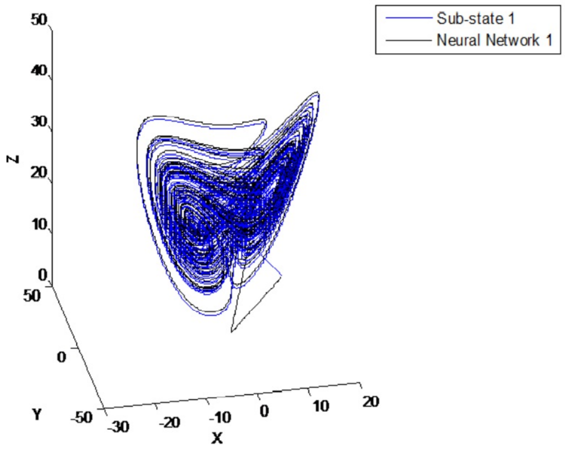

Fig. 3. Sub-State of Lorenz's attractor with initial condition X1 (0) = (10; 0; 10)ˆT

$$\begin{aligned} aD_t^{\alpha} x_{p_1} &= 10x_2 - 10x_1, \\ aD_t^{\alpha} x_{p_2} &= -x_2 - x_1 x_2 + 28x_1, \\ aD_t^{\alpha} x_{p_3} &= x_1 x_2 - \frac{8}{3} x_3, \\ x_i(0) &= (10, 0, 10)^T,\ i = 1, \end{aligned} \tag{23}$$

and the Chen's oscillator is described by:

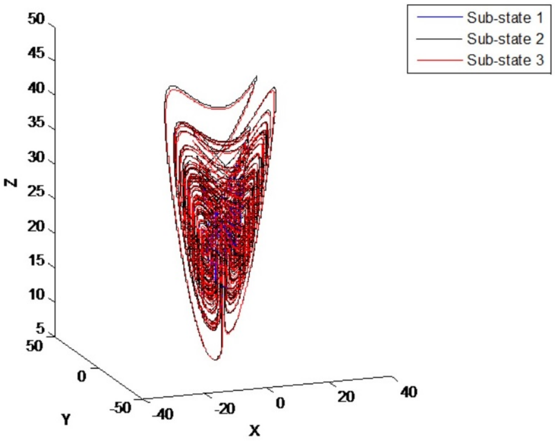

Fig. 4. Sub-States of Chen's attractor with initial condition X2,3,4(0) = (-10; 0; 37)ˆT

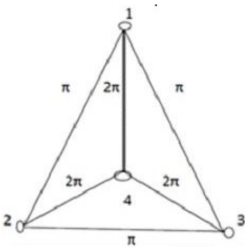

Fig. 5. Structure of the network with each node being a Lorentz and Chen's system

$$
\begin{aligned}
aD_t^{\alpha}x_{i_1} =& p_1(x_{i2}-x_{i1})+\sum_{j=1,j\neq i}^{4}c_{ij}a_{ij}(x_{j1}-x_{i1}),\\
aD_t^{\alpha}x_{i_2} =& (p_3-p_2)x_{i1}-x_{i1}x_{i3}+p_3x_{i2}+\\
& \sum_{j=1,j\neq i}^{4}c_{ij}a_{ij}(x_{j2}-x_{i2}),\\
aD_t^{\alpha}x_{i_3} =& x_{i1}x_{i2}-p_2x_{i3}+\sum_{j=1,j\neq i}^{4}c_{ij}a_{ij}(x_{j3}-x_{i3}),\\
x_i(0) =& (-10,0,37)^T,\ i=2,3,4.
\end{aligned} \tag{24}
$$

If the system parameters are selected as $p_1 = 35$, $p_2 = 3$, $p_3 = 28$, then the fractional order Lorenz's system and the fractional order Chen's system are shown in Fig. 1 and Fig.2, with $\alpha = \lambda = 1$, Fig. 3 and Fig.4, with $\alpha = \lambda = 0.0005$ respectively. In this set of system parameters, one unstable equilibrium point of the oscillator (25) is x = (7:9373; 7:9373; 21)T [12].

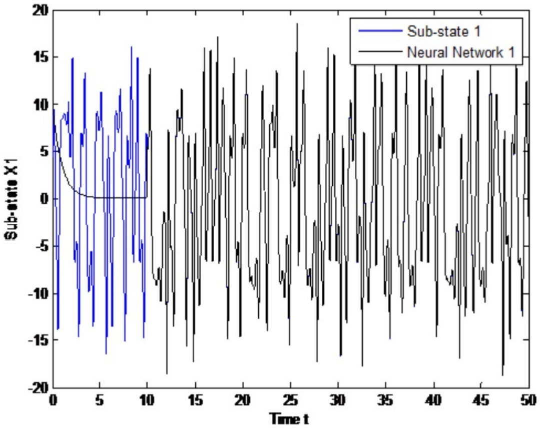

Fig. 6. Time evolution for sub-states 1 with initial state Xn1(0) = (10; 0; 10)ˆT

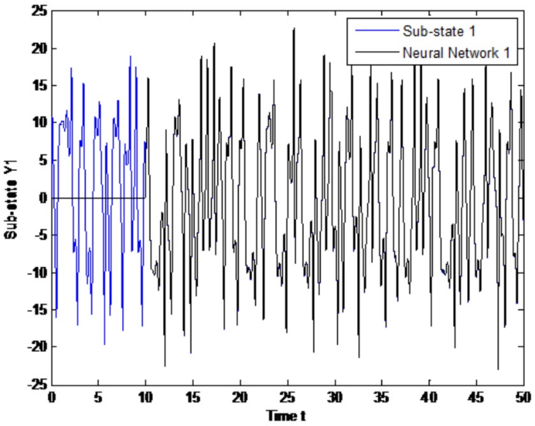

Fig. 7. Time evolution for sub-states 1 with initial state Xn1(0) = (10; 0; 10)ˆT

Suppose that each pair of two connected fractional order Lorenz and the fractional order Chen's oscillators are linked together through their identical sub-state variables, i.e.,$\Gamma = diag(1,1,1)$, and the coupling strengths are $c_{12} = c_{21} = \pi$, $c_{23} = c_{32} = \pi$, $c_{13} = c_{31} = \pi$, $c_{14} = c_{41} = 2\pi$, $c_{24} = c_{42} = 2\pi, c_{34} = c_{43} = 2\pi$. Fig. 5 visualizes this entire fractional order dynamical network.

The neural network was selected as:

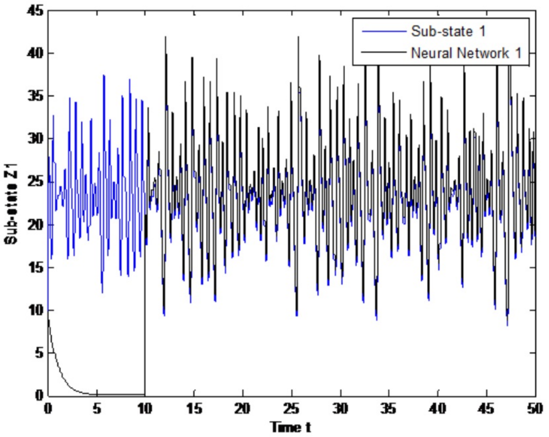

Fig. 8. Time evolution for sub-states 2 with initial state Xn1(0) = (10; 0; 10)ˆT

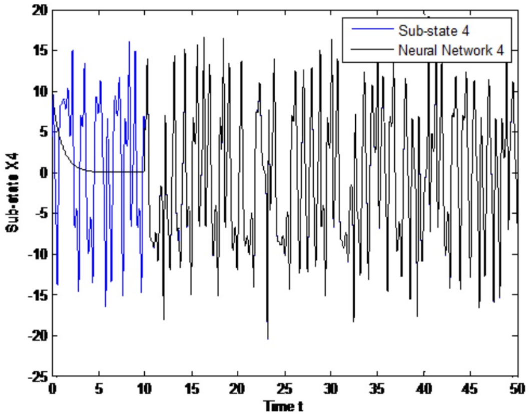

Fig. 9. Time evolution for sub-states 4 with initial state Xn4(0) = (20,20,-10)ˆT

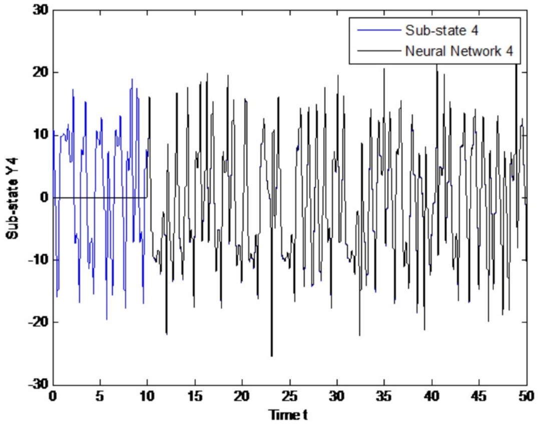

Fig. 10. Time evolution for sub-states 4 with initial state Xn4(0) = (20,20,-10)ˆT

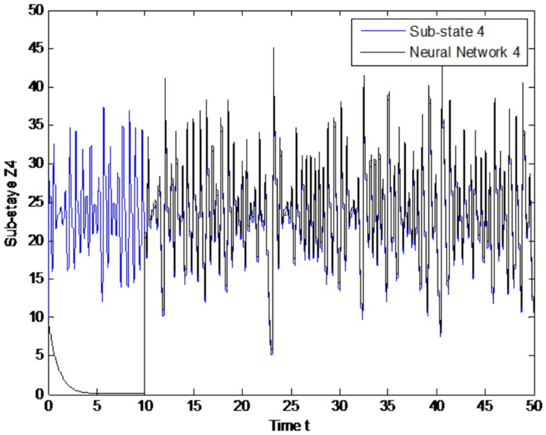

Fig. 11. Time evolution for sub-states 4 with initial state Xn4(0) = (20,20,-10)ˆT

$$A_{n_i} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$W_{n_i} = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 4 & 0 \\ 0 & 2 & 3 \end{pmatrix},$$

$$\sigma(x_{n_i}) = \begin{pmatrix} \tanh_{n_1}(x(t-\tau)) \\ \tanh_{n_2}(x(t-\tau)) \\ \tanh_{n_3}(x(t-\tau)) \end{pmatrix},$$

$$x_{n_i} = (20, 20, -10)^T,$$

$$L_{\phi_{\sigma_i}} \triangleq n_i = 3, \; i = 1, 2, 3, 4. \tag{25}$$

The experiment is performed as follows. Both systems, the recurrent neural network (2) and the dynamical networks (24) and (25), evolve independently; at that time $t = 10\ Seconds$, the proposed control law (22) is incepted. Simulation results are presented in Figs. 6-8, with $\alpha = \lambda = 1$, for sub-sates of node 1. As can be seen, tracking is successfully achieved and error is asymptotically stable, as it is shown in Figs. 9-11, with $\alpha = \lambda = 0.0005$ for sub-states of node.

6 Conclusions

We have presented a controller design for trajectory tracking of a fractional general complex dynamical networks. This framework is based on controlling dynamic neural networks using Lyapunov-Krasovskii theory in the fractional case. We obtained a control law in a purely theoretical way, and can be therefore to a wide range of problems in trajectory tracking.

The proposed control law guarantees the stability of the tracking error between plant and reference signals. The analytical results obtained that predict the stability of the tracking error between plant and reference signals are satisfactory, which can be seen through simulation, these clearly show the desired tracking.

As an example, the proposed control is applied to a simple network with four different nodes and five non-uniform links. In future work, we will consider the stochastic case in fractional systems.

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