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Toward Relevance Term Logic

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Abstract. Term Functor Logic is a term logic that recovers some important features of the traditional, Aristotelian logic; however, it turns out that it does not preserve all of the Aristotelian properties a valid inference should have insofar as its class of theorems includes some inferences that may be considered irrelevant. Given this situation, in this contribution we tweak a tableaux method in order to avoid said irrelevance.

Keywords. Semantic trees, term logic, relevance logic.

1 Introduction

Term Functor Logic is a logic that recovers some core features of the traditional, Aristotelian logic, mainly, its term syntax; however, as we will see, it turns out that it does not preserve all of the Aristotelian properties a full-blooded inference should have insofar as its class of theorems includes some inferences that may be considered irrelevant by the lights of the Aristotelian paradigm.

Given this situation, in this contribution we advance some tentative steps towards the creation of a relevance term logic. Hence, for a more detailed exposition of the family of term logics we are considering here [24, 11, 10, 20, 26, 14] and their tableaux, we refer the reader to our previous works [7, 3, 6, 5]; meanwhile, in order to achieve our present goal, we first provide a summary of some preliminary concepts and results (i.e. syllogistic, and Term Functor Logic and its tableaux), then we briefly explain the problem (how irrelevance is parasitic of Term Functor Logic) and, finally, we suggest a possible solution by tweaking a tableaux method.

2 Preliminaries

Syllogistic is a term logic that has its origins in Aristotle's *Prior Analytics* [1] and deals with inference using categorical statements. A *categorical statement* is a statement composed by two terms, a quantity, and a quality. The subject and the predicate of a statement are called *terms*: the term-schema S denotes the subject term of the statement and the term-schema P denotes the predicate. The *quantity* may be either universal (*All*) or particular (*Some*) and the *quality* may be either affirmative (*is*) or negative (*is not*). These categorical statements have a *type* denoted by a label (either a (universal affirmative, SaP), e (universal negative, SeP), i (particular affirmative, SiP), or o (particular negative, SoP)) that allows us to determine a *mood*, that is, a sequence of three categorical statements ordered in such a way that two statements are premises (major and minor) and the last one is a conclusion. A *categorical syllogism*, then, is a mood with three terms one of which appears in both premises but not in the conclusion. This particular term, usually denoted with the term-schema M, works as a link between the remaining terms and is known as the middle term. According to the position of this middle term, four *figures* can be set up in order to encode the valid syllogistic moods (Table1).[1]

This quick overview of syllogistic, though formally correct, is a little bit out of context. Syllogistic is an integral part of what we could call a basic *corpus aristotelicum* that, in turn, could be defined by the tuple $\mathfrak{A} = \langle Th_E, Th_C, Th_O, Th_P, Th_L \rangle$

[1] For sake of brevity, but without loss of generality, here we omit the syllogisms that require existential import.

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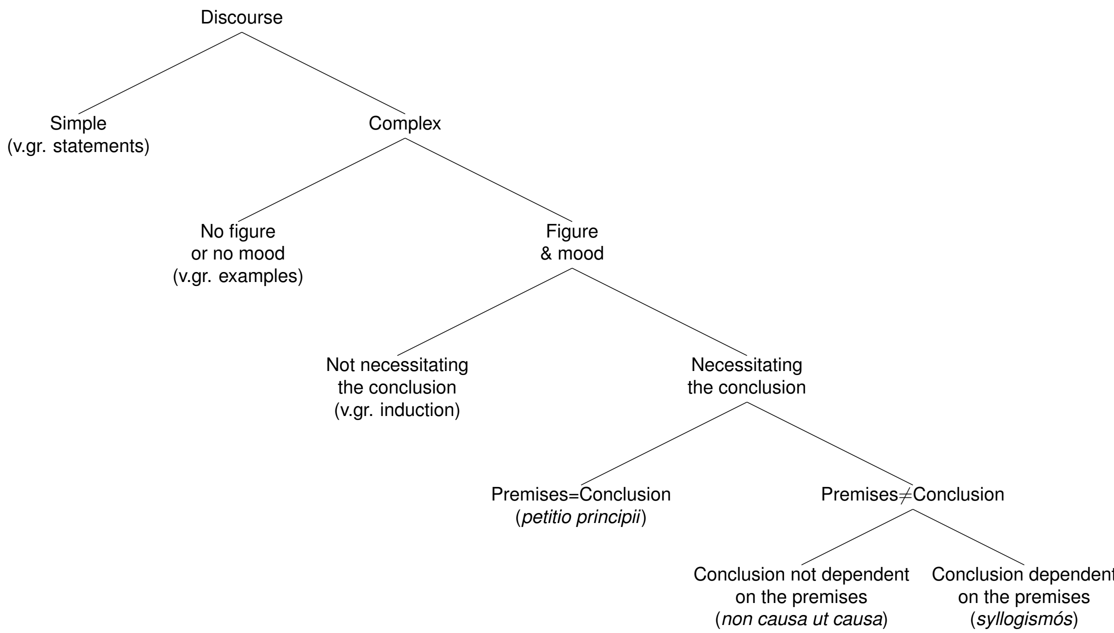

Fig. 1. The Boethian exposition of syllogistic (adapted from [27, p.44])

Table 1. Valid syllogistic moods by figure

First	Second	Third	Fourth
aaa	eae	iai	aee
eae	aee	aii	iai
aii	eio	oao	eio
eio	aoo	eio	

(cf. [16, p.4ff]) where Th_E is an epistemological theory that includes the production of hypothesis and inferences under the Aristotelian concepts of *epagogé* and *syllogismós*, respectively[2]; Th_C is a theory of causality that distinguishes material, formal, efficient, and final causes[3]; Th_O is an ontological theory that assumes a systemic view of the world given the double claim that there are no unhad properties (*contra* universals *ante rem*) nor objects without properties (*contra* bare particulars); Th_P is a psychological theory that makes good use of the concept “habit” in order to explain behavior (both *epagogé* and *syllogismós*, for example, would be habits when performed by agents); and Th_L is a logical theory designed for understanding categories, statements, inferences, explanations, and cognitive biases.

This last theory includes syllogistic as a theory of deductive inference but, as we have tried to imply, it has some specific semantic requirements related to the other components of the *corpus*. Hence, the formal description of syllogistic that we have given above lacks a quality that may be better understood given the previous context: syllogistic is a deductive theory designed to avoid causal irrelevance. In order to illustrate this last point consider Thom’s explanation of Kilwardby’s first exposition of syllogistic—also called the Boethian exposition (Figure 1).

[2]The concept *epagogé* refers to some sort of essential induction, so to speak, that is different from a numerical induction. This difference helps explain why some general statements are admissible (v.gr., *All human beings are living beings*) while others are not (v.gr., *All human beings are mexican*). The concept *syllogismós*, on the other hand, will be treated with more detail below.

[3]Notice this concept of cause is different, for example, from our current idea of a factor: two material factors may explain a state of affairs, and hence we may have a multi-factorial explanation of said state, but such an explanation needs not be a multicausal explanation, since both factors are instances of material causes.

This Boethian exposition clarifies that, within the Aristotelian way of thinking or paradigm, a syllogistic inference or syllogism—*syllogismós*—is a piece of complex discourse (insofar as it includes at least two premises and one conclusion) with mood and figure (because the order of statements and terms matters) in which a conclusion that is different from the premises (thus avoiding *petitio principii*) necessarily (and hence deductively) follows from and dependes on said premises (thus avoiding irrelevance, *non causa ut causa*).

This Aristotelian view of inference should not be understated because it differs from the contemporary, Fregean-Tarskian approach, at least in three respects: *i)* the contemporary approach takes it that content and form are independent (as when the usual logic handbooks claim, almost dogmatically, that logic does not deal with truth, but with validity), yet that independence is not crystal clear (cf. [2]); whereas in the Aristotelian approach content and form are systemic and codependent (as when Aristotle distinguishes between natural and unnatural predication (cf. [12, 13])). *ii)* The contemporary approach usually follows the Fregean paradigm that results from droping the ternary syntax of traditional logic (subject-copula-predicate) in order to promote a binary syntax (function-argument) imported from mathematics, which turns out to be not that natural (cf. [12, 28, 13]). And *iii)* the contemporary approach admits reflexivity (i.e. $p \vdash p$) both as a valid pattern of inference and, sometimes, as the principle of identity (i.e. $\vdash p \Rightarrow p$); whereas the Aristotelian approach rejects the former (i.e. $p \nvdash p$) while admits a version of the latter (i.e. $\vdash p \Rightarrow p$). We will return to some of these issues later.

2.1 Term Functor Logic and its Tableaux

Term Functor Logic (TFL, for short) [24, 26, 9, 11, 14] is a plus-minus algebra that employs terms and functors rather than first order language elements such as individual variables or quantifiers (cf. [23, 21, 15, 24, 25, 19]). According to this algebra, the four categorical statements can be represented by the following syntax [11]:

a. $\mathsf{SaP} := -\mathsf{S} + \mathsf{P}$,

Table 2. A valid syllogism

	Statement	TFL
1.	All computer scientists are animals.	$-\mathsf{C} + \mathsf{A}$
2.	All logicians are computer scientists.	$-\mathsf{L} + \mathsf{C}$
$\vdash$	All logicians are animals.	$-\mathsf{L} + \mathsf{A}$

b. $\mathsf{SeP} := -\mathsf{S} - \mathsf{P}$,

c. $\mathsf{SiP} := +\mathsf{S} + \mathsf{P}$,

d. $\mathsf{SoP} := +\mathsf{S} - \mathsf{P}$.

Given this representation, TFL provides a simple rule for syllogistic inference: a conclusion follows validly from a set of premises if and only if *i)* the sum of the premises is algebraically equal to the conclusion and *ii)* the number of conclusions with particular quantity (viz., zero or one) is the same as the number of premises with particular quantity [11, p.167]. Thus, for instance, if we consider a valid syllogism (say, a syllogism aaa of the first figure, aaa-1), we can see how the application of this rule produces the right conclusion (Table 2).

In this example we can clearly see how the rule works: *i)* if we add up the premises we obtain the algebraic expression $(-\mathsf{C} + \mathsf{A}) + (-\mathsf{L} + \mathsf{C}) = -\mathsf{C} + \mathsf{A} - \mathsf{L} + \mathsf{C} = -\mathsf{L} + \mathsf{A}$, so that the sum of the premises is algebraically equal to the conclusion and the conclusion is $-\mathsf{L} + \mathsf{A}$, rather than $+\mathsf{A} - \mathsf{L}$, because *ii)* the number of conclusions with particular quantity (zero in this case) is the same as the number of premises with particular quantity (zero in this case)[4]. In contrast, for sake of comparison, consider an invalid syllogism (aaa-3) that does not add up (Table 3).

Now, as exposed in [7, 4] and following [8, 22], we can develop a tableaux proof method for TFL. Hence, we say a *tableau* for TFL is an acyclic connected graph determined by nodes and vertices. The node at the top is called *root*. The nodes at the bottom are called *tips*. Any path from

[4]Although we are exemplifying this logic with syllogistic inferences, this system is capable of representing relational, singular, and compound inferences with ease and clarity. Furthermore, TFL is arguably more expressive than classical first order logic [9, p.172].

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Table 3. An invalid syllogism

	Statement	TFL
1.	All computer scientists are animals.	$-C+A$
2.	All computer scientists are logicians.	$-C+L$
$\nvdash$	All logicians are animals.	$-L+A$

Fig. 2. TFL expansion rules

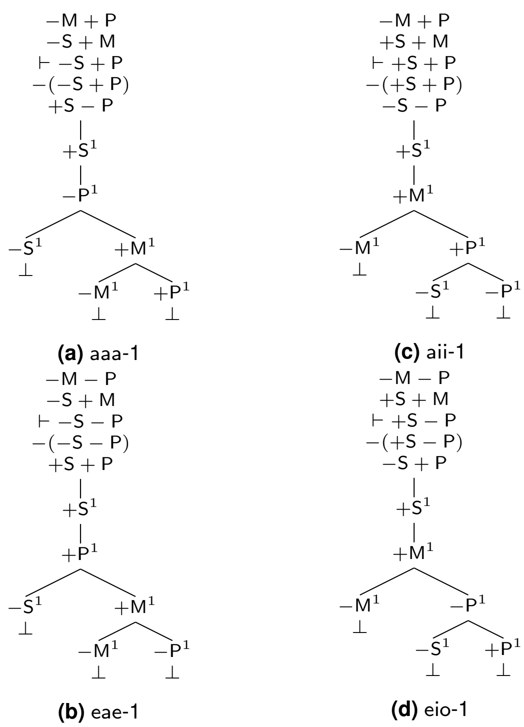

Fig. 3. Valid syllogistic moods of the first figure

the root down a series of vertices is a *branch*. To test an inference for validity we construct a tableau which begins with a single branch at whose nodes occur the premises and the rejection of the conclusion: this is the *initial list*. We then apply the rules that allow us to extend the initial list (Figure 2).

Figure 2a depicts the rule for a (e) statements, while Figure 2b shows the rule for i (o) statements. After applying a rule we introduce some index $i \in \{1, 2, 3, \ldots\}$. For statements a and e, the index may be any natural; for statements i and o, the index has to be a new natural if they do not already have an index. Also, following TFL tenets, we assume the following rules of rejection: $-(\pm T) = \mp T$, $-(\pm T \pm T) = \mp T \mp T$, and $-(--T--T) = +(-T)+(-T)$.

A tableau is *complete* if and only if every rule that can be applied has been applied. A branch is *closed* if and only if there are terms of the form $\pm T^i$ and $\mp T^i$ on two of its nodes; otherwise it is *open*. A closed branch is indicated by writing a $\perp$ at the end of it; an open branch is indicated by writing ∞. A tableau is *closed* if and only if every branch is closed; otherwise it is *open*. So, as usual, $\pm T$ is a logical consequence of the set of terms Γ (i.e. $\Gamma \vdash \pm T$) if and only if there is a closed complete tableau whose initial list includes the terms of Γ and the rejection of $\pm T$ (i.e. $\Gamma \cup \{\mp T\} \vdash \perp$). Accordingly, we provide some examples (Figure 3).

To describe the process we follow to unfold each tableaux consider Figure 3a (cf. [4]). The first three lines are the premises and the conclusion, and the fourth line is the rejection of the conclusion: all these lines but the conclusion define the initial list. Then the fifth line is the result of applying a rule of rejection to the conclusion. Then the next couple of lines is the result of applying the rule for an i proposition to the fifth line, picking index 1. Then the first split results from applying the rule for an a proposition to the second line (i.e. the minor premise), also picking index 1, since we want the indexes to unify. This split produces two branches, one of which (the leftmost) includes terms $+S^1$ and $-S^1$ on two of its nodes, and hence is closed; the remaining branch is not closed yet, so we continue with the same process: we split the last available premise (i.e. the major premise) to obtain, again, a couple of branches, one of which (the leftmost) includes terms $-M^1$ and $+M^1$ on two of its nodes, and hence is closed; and the other (the rightmost) that contains terms $+P^1$ and $-P^1$ on two of its nodes, and hence is closed as well.

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Table 4. Some problematic inferences

I	II	III	IV
1.$-A+B$	1.$\pm B$	1.$-A+B$	1. $\pm A$
2.$+A-B$	$\vdash -A+A$	2.$-C+A$	$\vdash \pm A$
$\vdash -A+A$		$\vdash -C+A$	

Fig. 4. RTL expansion rules

3 Toward Relevance Term Logic

At this point it should be clear that TFL recovers some syntactical features of the traditional, Aristotelian logic, particularly, a term syntax; however, it turns out that it does not preserve all of the Aristotelian properties a proper inference should have because its class of theorems includes some inferences that can be considered irrelevant by the lights of the Boethian exposition and the Aristotelian paradigm. In order to exemplify this issue consider the problematic inferences shown in Table 4.

Such inferences are problematic because all of them are valid in TFL (cf. [26]) (as well as in classical First Order Logic, we should add), and yet, they cannot be valid within an Aristotelian framework: inference I is a case of *ex contradictione sequitur quodlibet* (ECSQ)—i.e. a contradiction implies anything—; inference II is an instance of the (positive) paradox of implication—i.e. a tautology is implied by anything—; inferences III and IV are instances of *petitio principii*. But then there is an impasse: while TFL is close to an Aristotelian notion of inference (given its syntatical features), it is still far from being a relevance logic in an Aristotelian sense (since irrelevance is parasitic of TFL). To solve this deadlock, consider the proposal given in Figure 4.

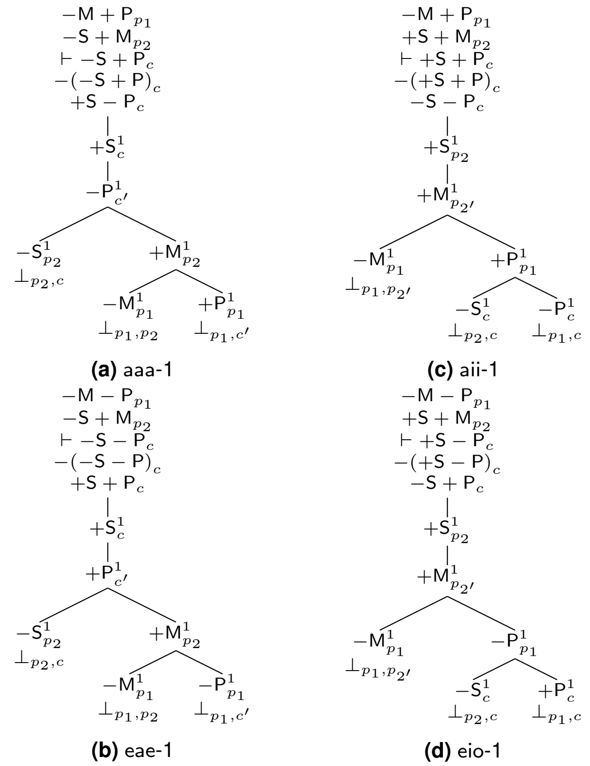

Fig. 5. Valid syllogistic moods within the first figure (again)

These Relevance Term Logic (RTL) tableaux rules behave as the tableaux rules for TFL, but notice that besides the indexes, we introduce and keep a flag $f, f' \in \{p_i, c\}$ for $i \in \{1, 2, 3, \dots\}$ (p for premise, c for conclusion). Now we say a branch is *open* if and only if there are no terms of the form $\pm T^i$ and $\mp T^i$ on it; a branch is *semi-open* (or *semi-closed*) if and only if there are terms of the form $\pm T^i_f$ and $\mp T^i_f$; otherwise it is *closed*. An open branch is indicated by writing ∞ at the end of it; a semi-open (semi-closed) branch is indicated by writing $\propto_{f,f}$ ($\infty_{f,f}$); and a closed branch, as usual, is denoted by $\perp_{f,f'}$. We say a tableau is *Aristotelian* if and only if every branch is closed and all the flags are carried at the end of every tip; a tableau is *open* if and only if it has an open branch; otherwise, it is *classical*. The rest of definitions is as usual.

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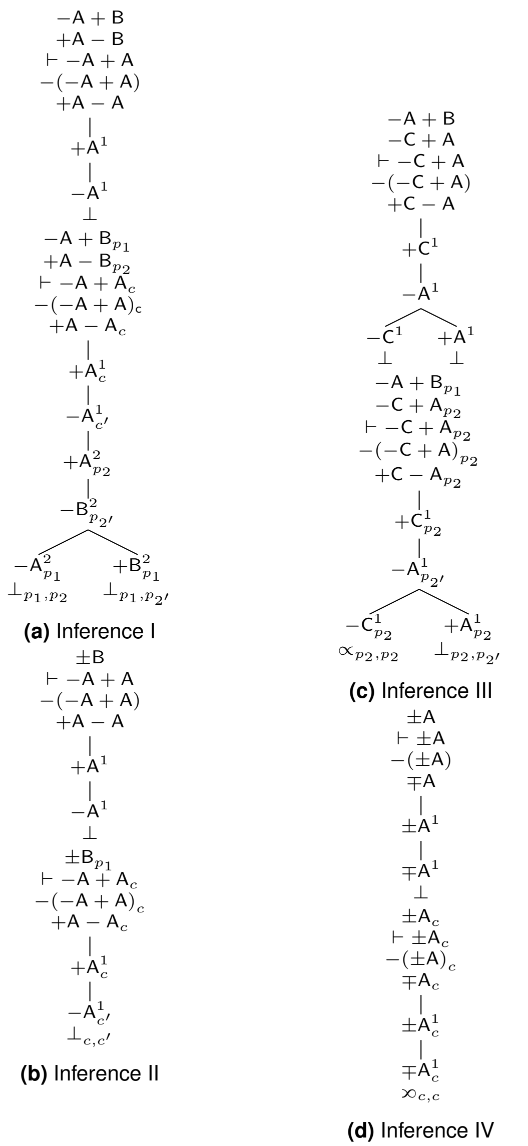

(a) Inference I
(b) Inference II
(c) Inference III
(d) Inference IV

Fig. 6. Some problematic inferences: TFL (above) vs RTL (below)

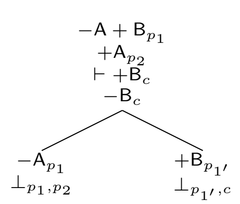

Fig. 7. *Modus Ponens*

Accordingly, reconsider and compare the basic syllogistic moods—they are correct both in TFL and in RTL (Figure 5)—and the problematic inferences shown in Table 4 above—even though they are classically valid, they are not Aristotelian (Figure 6).

We think the examples shown in Figure 5 are self-explanatory, but perhaps a brief description of Figure 6 may help explain further the use of these rules. So, Figure 6a shows an instance of ECSQ. We can see that the TFL tree is just closed, whereas the RTL tree is also closed but is not Aristotelian because the closure does not use any conclusion (i.e. the premises are not relevant to the conclusion). Figure 6b shows an instance of a paradox of implication and, while the TFL tree is just closed, the RTL tree is closed but not all the flags are carried to the tips, and hence the conclusion is not relevant to the premise. Figure 6c and 6d show instances of *petitio principii*: observe that while the corresponding TFL trees are closed, the RTL trees are semi-open (semi-closed) because the closure does not use the conclusion or the premises (i.e. the conclusion is not relevant to the premises or vice versa). This means that these inferences, although truth preserving, are not relevant; and hence, while they are not to be regarded as full-blooded inferences, they should not be discarded altogether as totally wrong inferences.

Additionally consider, just out of curiosity, some inferences in order to suggest that this proposal seems to be suitable for non-syllogistic logic. Take an instance of a *Modus Ponens* for propositional logic (Figure 7), and take an instance of a relational inference (say, "since every B loves some G and every G is W, it follows that every B loves something W") for relational logic (Figure 8). This particular examples would suggest said inferences are not only classically valid or truth preserving, but also relevant in an Aristotelian sense.

Our claim, thus, is that this proposal moves TFL into the direction of a relevance logic that is skeptical of both *petitio principii* and *non causa ut causa* inferences. So, in a sense, we are saying that:

Theorem 1 *RTL is Aristotelian.*

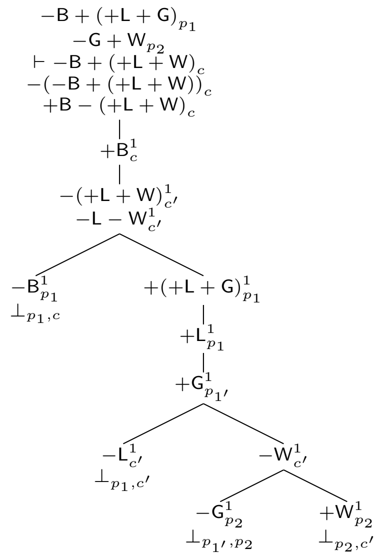

Fig. 8. A relational inference

Indeed, *i)* RTL is wary of *petitio principii* (i.e. instances of inferences such as III and IV). Aristotle suggested that a *petitio principii* is a fallacy because it fails to account for a causal explanation since it depends upon assuming what has to be explained (*De Sophisticis Elenchis* 168b23-27). It is a requirement of a legitimate inference that the conclusion (i.e. what has to be explained) has to be different from the premises (*Topics* 100a25-26, *De Sophisticis Elenchis* 165a1-2, *Prior Analytics* 24b19-20).

ii) RTL is wary of *non causa ut causa* (i.e. instances of inferences such as I and II). Contemporary, classical First Order Logic admits both the rule ECSQ and the paradoxes of implication as patterns of valid inference, but this view allows some sort of irrelevance that Aristotle did not quite accept (*Prior Analytics* 2 4-57b3): this sort of irrelevance, as we have seen, is parasitic of TFL as well.

iii) Finally, RTL avoids transforming the First Principle (i.e. the identity principle) into the First Fallacy (i.e. *petitio principii*), as [18] would put it (inference IV)—of course, our proposal is far from being as sophisticated as theirs, but we believe it could be useful.

4 Final remarks

As we have tried to show, Term Functor Logic is an alternative logic that recovers some important features of the traditional, Aristotelian logic but, as we have seen, it does not preserve all of the Aristotelian properties a proper inference should have insofar as its class of theorems includes some inferences that may be considered irrelevant. Since this situation is problematic, in this contribution we have offered a preliminary, provisional tableaux method for a relevance logic version of Term Functor Logic; nevertheless, given the current scope and the space limitations of this tentative research, we believe our immediate challenges include, at least: *i)* checking the (in)validity of more (non-)problematic inferences and looking for soundness and completeness; *ii)* offering a cogent, philosophical interpretation of the proposed method (say, in terms of *propter quid* and *quia* inferences); *iii)* reverse engineering the rules of RTL into TFL; and *iv)* further discussing the place of this proposal, if any, within the current literature about relevance logic (cf. [17]). We are currently working on these issues.

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References

1. **Aristotle (1989).** Prior Analytics. Hackett Classics Series. Hackett.
2. **Cabrera, J. (2003).** Es realmente la lógica tópicamente neutra y completamente general. Ergo, Nueva Época, Revista de Filosofía, Vol. 12, pp. 7–33.
3. **Castro-Manzano, J. M. (2019).** Silogística intermedia, términos y árboles. Tópicos, Revista De Filosofía, Vol. 58, pp. 209–237.
4. **Castro-Manzano, J.-M. (2020).** Distribution tableaux, distribution models. Axioms, Vol. 9, No. 2.

ISSN 2007-9737

5. **Castro-Manzano, J. M. (2020).** Murphree's numerical term logic tableaux. Electronic Notes in Theoretical Computer Science, Vol. 354, pp. 17–28. Proceedings of the Eleventh and Twelfth Latin American Workshop on Logic/Languages, Algorithms and New Methods of Reasoning (LANMR).

6. **Castro-Manzano, J. M. (2020).** Un método de árboles para la silogística modal. Open Insight, Revista De Filosofía, Vol. 58, pp. 209–237.

7. **Castro-Manzano, J. M., Reyes-Cardenas, P.-O. (2018).** Term functor logic tableaux. South American Journal of Logic, Vol. 4, No. 1, pp. 9–50.

8. **D'Agostino, M., Gabbay, D. M., Hähnle, R., Posegga, J. (1999).** Handbook of Tableau Methods. Springer.

9. **Englebretsen, G. (1987).** The New Syllogistic. P. Lang.

10. **Englebretsen, G. (1988).** Preliminary notes on a new modal syllogistic. Notre Dame J. Formal Logic, Vol. 29, No. 3, pp. 381–395.

11. **Englebretsen, G. (1996).** Something to Reckon with: The Logic of Terms. Canadian electronic library: Books collection. University of Ottawa Press.

12. **Englebretsen, G. (2013).** Robust Reality: An Essay in Formal Ontology. Philosophische Analyse / Philosophical Analysis. De Gruyter.

13. **Englebretsen, G. (2017).** Bare Facts and Naked Truths: A New Correspondence Theory of Truth. Taylor & Francis.

14. **Englebretsen, G., Sayward, C. (2011).** Philosophical Logic: An Introduction to Advanced Topics. Bloomsbury Academic.

15. **Kuhn, S. T. (1983).** An axiomatization of predicate functor logic. Notre Dame J. Formal Logic, Vol. 24, No. 2, pp. 233–241.

16. **Losee, J. (2001).** A Historical Introduction to the Philosophy of Science. OUP Oxford.

17. **Mares, E. (2004).** Relevant Logic: A Philosophical Interpretation. Cambridge University Press.

18. **Meyer, R., Martin, E. (2019).** S (for syllogism) revisited. The Australasian Journal of Logic, Vol. 16, No. 3, pp. 49–67.

19. **Moss, L. (2015).** Natural logic. In **Lappin, S., Fox, C.**, editors, The Handbook of Contemporary Semantic Theory. John Wiley & Sons.

20. **Murphree, W. A. (1998).** Numerical term logic. Notre Dame J. Formal Logic, Vol. 39, No. 3, pp. 346–362.

21. **Noah, A. (1980).** Predicate-functors and the limits of decidability in logic. Notre Dame J. Formal Logic, Vol. 21, No. 4, pp. 701–707.

22. **Priest, G. (2008).** An Introduction to Non-Classical Logic: From If to Is. Cambridge Introductions to Philosophy. Cambridge University Press.

23. **Quine, W. V. O. (1971).** Predicate functor logic. **Fenstad, J. E.**, editor, Proceedings of the Second Scandinavian Logic Symposium, North-Holland.

24. **Sommers, F. (1982).** The Logic of Natural Language. Clarendon Library of Logic and Philosophy. Oxford University Press.

25. **Sommers, F. (2005).** Intelectual autobiography. In **Oderberg, D. S.**, editor, The Old New Logic: Essays on the Philosophy of Fred Sommers. Bradford book, pp. 1–24.

26. **Sommers, F., Englebretsen, G. (2000).** An Invitation to Formal Reasoning: The Logic of Terms. Ashgate.

27. **Thom, P. (2007).** Logic and Ontology in the Syllogistic of Robert Kilwardby. Studien Und Texte Zur Geistesgeschichte Des Mittelalters. Brill.

28. **Woods, J. (2016).** Logic Naturalized. Springer International Publishing, Cham, pp. 403–432.

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