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Abstract. The Industry 4.0 paradigm aims to obtain high levels of productivity and efficiency, more competitive final products and compliance with the demanding regulations related to industrial safety. To achieve these objectives, the industrial systems must be equipped with condition monitoring systems for early detection, isolation, and location of faults. The paper presents a proposal for a condition monitoring system characterized by its robustness in presence of noise and missing variables in the measurements. The proposal combines the use of simple and effective imputation algorithms with a fuzzy classification kernel algorithm based on the use of the non-standard Pythagorean fuzzy sets. The proposed scheme was validated using the known DAMADICS test problem with excellent results.

Keywords. Robust condition monitoring, Pythagorean fuzzy sets, missing information, noise.

1 Introduction

Currently, terms such as smart industry and Industry 4.0 paradigm are very common in the scientific literature related to the automation of industrial plants. They aim to obtain high productivity levels, more competitive final products, and excellent compliance of the industrial safety standards. To achieve these objectives, the industrial systems must be equipped with condition monitoring systems for early detection, isolation, and location of faults that cause safety problems, economic loss, and environmental effects. This goal has guided a large amount of research on the topic of condition monitoring in industrial plants in the last twenty years [10, 25].

The methodologies for condition monitoring are divided into two groups [25]. The first group encompasses the methods based on the process models [6, 8].

The successful use of these methodologies depends on a deep understanding of the process, its operational parameters, and operating modes. However, this is very difficult to achieve due to the high complexity of current industrial systems.

The second group includes the data-driven methods, which do not need an accurate mathematical model, and initial deep knowledge of the process parameters [15].

Among the most used techniques in this group are the computational intelligence tools based on: fuzzy logic [9], clustering [19, 22], statistical tools [24] and artificial neural networks [17, 18] just to mention some of them.

The data-driven methodologies are seriously affected by noise and missing variables in measurements, which is usual to find in industrial plants. This indicates the necessity to consider these factors for obtaining satisfactory

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Fig. 1. Classification scheme for condition monitoring with noise and missing variables in the measurements

Algorithm ⁻	1:	Kernel	Pythagorean	Fuzzy
C-Means alg	or	ithm (KF	PyFCM)	
Data: I, γ :	> (0, m > 1	, δ , Itr_{max} (ma	aximum

number of iterations) **Result:** fuzzy partition U, class centers V 1 Initialize U to random fuzzy partition; 2 $Itr \leftarrow 1$;

- з repeat
- 4 Update the class centers V according to (17);
- 5 Calculate the distances according to (13);
- 6 Update U according to (16).;

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7 Itr \leftarrow Itr + 1;
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s until $||U_t - U_{t-1}|| < \gamma \wedge ltr \geq Itr_{max};$

performances of the condition monitoring systems [12, 13]. A review of the scientific literature shows that in most papers the robustness versus noise and missing variables in the observations are not analyzed together.

Missing data in the observations can be due to several causes as sensor faults and problems in the operation of the data acquisition systems, just to mention some of them.

Missing data can be very important to find deviations not allowed in the operation of an industrial process.

Computación y Sistemas, Vol. 27, No. 1, 2023, pp. 223–235 doi: 10.13053/CyS-27-1-4534 For this reason, it is recommended to use imputation methods for their treatment [2, 12]. The non-standard Pythagorean fuzzy sets were presented in [27, 26], where it is shown that the space of Pythagorean membership grades is greater than the space of intuitionistic membership grades.

This represents an important advantage in condition monitoring because it allows the insertion of the uncertainty in the specification of membership degrees as result of noisy measurements and the data imputation.

The aim of this paper is to propose a robust condition monitoring scheme versus noise and missing variables in the observations obtained from an industrial plant. For this, it is presented a hybrid scheme that combines simple and effective imputation algorithms with Pythagorean fuzzy set tools.

In the methodology, the imputation of the missing variables in the observation obtained from the industrial plant is performed online to later get classified. The Pythagorean Fuzzy C-Mean algorithm (PyFCM), and its kernel version (KPyFCM) are used.

The proposal presents a high performance in presence of noise and the possible deviations that can be introduced in the imputation of the missing data.

Operation							
Mode	Description	Variable	Description				
NO	Normal Operation	CV	Process control external signal				
F1	Valve clogging	P1	Pressure on inlet valve				
F7	Critical flow	P2	Pressure on outlet valve				
F12	Electro-pneumatic transducer fault	Х	Valve plug displacement				
F15	Positioner spring fault	F	Main pipeline flow rat				
F19	Flow rate sensor fault	PV	Process value				

Table 1. Operation modes and measured variables in DAMADICS

Table 2. Characteristics of the training database

Parameter	Description	Quantity
l	Operation Modes (Classes)	6
p	Measured Variables	6
k	Number of Observations per class	1000
n	Number of Observation in the training database	6000

2 Materials and Methods

2.1 Missing Data Imputation

The information loss has been modeled according to three main mechanisms: Missing Completely at Random (MCAR), Missing at Random (MAR), and Missing Not at Random (MNAR)[11].

In this paper will be used MCAR mechanism which is the one mainly occurring in industrial plants [12].

In the scientific information, several techniques for data imputation have been presented [2].

However, all of them make the imputation using the entire data set or part of it without considering time requirements. In most control loops in industrial plants, the time requirements are very important.

For this reason, the imputation of the missing variables must be done online for each observation obtained from the plant satisfying strict time requirements.

Furthermore, it is necessary to take into account the noise that affects the measurements in most industrial processes. The imputation methods used in this paper are the arithmetic mean and the mode [11]. Their effectiveness and easy implementation were considered for their selection.

2.2 General Characteristics of the Pythagorean Fuzzy Sets (PFS)

In [27, 26], the PFS were introduced. The membership grades associated with the PFS will be called Pythagorean membership grades (PMG) and they can be expressed as follow:

- Two values, r(z) and d(z), are assigned for each $z \in Z$.
- If $r(z) \in [0,1]$, it is called strength of commitment at z
- If $d(z) \in [0,1]$, it is called direction of commitment at z.
- $\mathcal{H}_Y(z)$ is a membership grade which indicates the support for membership of z in \mathcal{H} .
- $\mathcal{H}_N(z)$ is a membership grade which indicates the support against membership of z in \mathcal{H} .

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Fig. 2. Faults classification (%) for DAMADICS process (Without missing variables)



Fig. 3. Faults classification (%) for DAMADICS process (1 randomly missing variable, imputation with mean values)

 $- \mathcal{H}_{Y}(z)$ and $\mathcal{H}_{N}(z)$ are defined as:

$$\mathcal{H}_Y(z) = r(z)\cos(\varphi(z)),\tag{1}$$

$$\mathcal{H}_N(z) = r(z)\sin(\varphi(z)), \tag{2}$$

where:

$$\varphi(z) = (1 - d(z))\frac{\pi}{2}, \qquad (3)$$

and $\varphi(z) \in [0, \frac{\pi}{2}]$ is expressed in radians.

Lemma: $\mathcal{H}_Y(z)$ and $\mathcal{H}_N(z)$ are Pythagorean complements with respect to r(z).

Proof: Squaring Eqs. (1) and (2).

$$\mathcal{H}_Y^2(z) = r^2(z)\cos^2(\varphi(z)), \tag{4}$$

$$\mathcal{H}_N^2(z) = r^2(z) \sin^2(\varphi(z)), \tag{5}$$

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and, by adding both equations:

$$\mathcal{H}_{Y}^{2}(z) + \mathcal{H}_{N}^{2}(z) = r^{2}(z)(\cos^{2}(\varphi) + \sin^{2}(\varphi)).$$
 (6)

From the Pythagorean theorem, it is known that $\cos^2(\varphi) + \sin^2(\varphi) = 1$. So that:

$$\mathcal{H}_Y^2(z) + \mathcal{H}_N^2(z) = r^2(z) \underbrace{(\cos^2(\varphi) + \sin^2(\varphi))}_{1}, \quad (7)$$

and hence:

$$\mathcal{H}_Y^2(z) = r^2(z) - \mathcal{H}_N^2(z). \tag{8}$$

Thus, it is evident that \mathcal{H}_Y and \mathcal{H}_N are Pythagorean complements with respect to $r(z) \diamondsuit$

In a general form, a PMG is represented by a pair of values (e,f) such that $e,f\in[0,1]$ and e^2 + $f^2\leq$ 1.



Fig. 4. Faults classification (%) for DAMADICS process (Random number of missing variables between 0 and 2, imputation with mean values)



Fig. 5. Faults classification (%) for DAMADICS process (1 randomly missing variable, imputation with mode values)

In this case, $e = \mathcal{H}_Y(z)$, indicates the degree of support for membership of z in \mathcal{H} and, $f = \mathcal{H}_N(z)$ indicates the degree of support against membership of z in \mathcal{H} .

Considering the pair (e, f), the equation (6) can be expressed as $e^2 + f^2 = r^2$.

This indicates that a PMG is a point of a circle of radius r. Intuitionistic membership grades are also represented by a pair (e, f) which satisfies $e, f \in [0, 1]$ and $e + f \leq 1$ [3].

Theorem: The set of Pythagorean membership grades is greater than the set of intuitionistics membership grades.

Proof: See [1].

The most significant consequence of this result is the possibility of using Pythagorean fuzzy sets in more situations than intuitionistics fuzzy sets. For condition monitoring applications, this characteristic of the PFS is very important for reducing classification errors.





Fig. 6. Faults classification (%) for DAMADICS process (Random number of missing variables between 0 and 2, imputation with mode values)

Table 3. Values of TP, FN, TN and FP for each of the analyzed faults (random number of missing variables between 0 and 2; imputation with mean value

Whithout noise				29	2% Noise Level 5%			o Noise Level				
Fault	TP	FN	TN	FP	TP	FN	ΤN	FP	TP	FN	ΤN	FP
1	193	7	785	0	186	14	754	4	183	17	737	9
7	198	2	780	6	193	7	747	10	188	12	732	15
12	198	2	780	3	191	9	749	6	185	15	735	10
15	192	8	786	9	180	20	760	23	177	23	743	27
19	197	3	781	4	190	10	750	16	187	13	733	20

2.3 Pythagorean Fuzzy C-Means Algorithm (PyFCM)

Considering the theory of PFS, the objective function on the PyFCM algorithm is similar to the one obtained for the Intuitionistic Fuzzy C-Means algorithm (IFCM) [3] according to the equation:

$$J_{\rm PyFCM} = \sum_{i=1}^{l} \sum_{k=1}^{N} u_{ik}^{*m} d_{ik}^2 + \sum_{i=1}^{l} \pi_i^* e^{1-\pi^*}, \qquad (9)$$

where m > 1 is the fuzziness regulation factor of the partition [21], l is the number of classes, N is the number of observations.

 $u_{ik}^* = u_{ik}^m + \pi_{ik}$. u_{ik}^* denotes the pythagorean fuzzy membership, u_{ik} represents the typical fuzzy membership of the *kth* observation in the *ith*

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class, and π_{ik} is the hesitation degree, which is defined as:

$$\pi_{ik} = 1 - u_{ik}^2 - (1 - u_{ik}^{\alpha})^{2/\alpha}, \alpha > 0,$$
 (10)

and

$$\pi_i^* = \frac{1}{N} \sum_{k=1}^N \pi_{ik}, k \in [1, N].$$
(11)

2.4 Kernel Pythagorean Fuzzy C-Means Algorithm

Kernel functions are used for mapping non-linear data from the input space to a higher-dimensional space.

	Whithout noise		2 % No	ise Level	5% Noise Level		
Fault	Sensitivity	1-Specificity	Sensitivity	1-Specificity	Sensitivity	1-Specificity	
1	96.50	0	93	0.53	91.50	1.21	
7	99.00	0.76	96.50	1.32	94.00	2.01	
12	99.00	0.38	95.50	0.79	92.50	1.34	
15	96.00	1.13	90.00	2.94	88.50	3.51	
19	98.50	0.51	95.00	2.09	93.50	2.66	

Table 4. Results in % of the Sensitivity and (1-Specificity)

This is very useful in classification tasks because it allows to achieve greater separability among classes, and reduce the classification errors.

In order to enhance the classification process, the kernel version of the PyFCM (KPyFCM) is obtained. In the KPyFCM algorithm the following objective function is minimized:

$$J_{\text{KPyFCM}} = \sum_{i=1}^{l} \sum_{k=1}^{N} u_{ik}^{*m} \| \Psi(\mathbf{z}_{k}) - \Psi(\mathbf{v}_{i}) \|^{2} + \sum_{i=1}^{l} \pi_{i}^{*} e^{1-\pi^{*}}, \qquad (12)$$

where $\|\Psi(\mathbf{z}_k) - \Psi(\mathbf{v}_i)\|^2$ is the square of the distance between $\Psi(\mathbf{z}_k)$ and $\Psi(\mathbf{v}_i)$. The distance in the feature space is calculated through the kernel in the input space as follows:

$$\begin{split} \left\| \Psi(\mathbf{z}_{\mathbf{k}}) - \Psi(\mathbf{v}_{\mathbf{i}}) \right\|^2 &= \mathbf{K}(\mathbf{z}_{\mathbf{k}}, \mathbf{z}_{\mathbf{k}}) - 2\mathbf{K}(\mathbf{z}_{\mathbf{k}}, \mathbf{v}_{\mathbf{i}}) \\ &+ \mathbf{K}(\mathbf{v}_{\mathbf{i}}, \mathbf{v}_{\mathbf{i}}). \end{split} \tag{13}$$

It is possible to find many different kernel functions in the scientific information. Nonetheless, the Gaussian kernel is one of the most popular.

In general, the choice of a function kernel depends on the application [5, 16]. If the Gaussian kernel function is used, then $\mathbf{K}(\mathbf{z}, \mathbf{z}) = 1$ and $\|\Psi(\mathbf{z}_k) - \Psi(\mathbf{v}_i)\|^2 = 2(1 - \mathbf{K}(\mathbf{z}_k, \mathbf{v}_i))$. So, Eq. (12) can be expressed as:

$$J_{\text{KPyFCM}} = 2\sum_{i=1}^{l} \sum_{k=1}^{N} u_{ik}^{*m} \|1 - \mathbf{K}(\mathbf{z}_{k}, \mathbf{v}_{i})\|^{2} + \sum_{i=1}^{l} \pi_{i}^{*} e^{1 - \pi^{*}}, \qquad (14)$$

where

$$\mathbf{K}(\mathbf{z}_{\mathbf{k}}, \mathbf{v}_{\mathbf{i}}) = e^{-\|\mathbf{z}_{k} - \mathbf{v}_{i}\|^{2}/\delta^{2}},$$
(15)

where δ is the bandwidth which indicates the smoothness degree of the Gaussian kernel function [21]. Minimizing Eq. (14), yields:

$$u_{ik}^* = \frac{1}{\sum_{j=1}^l \left(\frac{1-\mathbf{K}(\mathbf{z}_k, \mathbf{v}_i)}{1-\mathbf{K}(\mathbf{z}_k, \mathbf{v}_j)}\right)^{1/(m-1)}}, \quad (16)$$

$$\mathbf{v}_{i} = \frac{\sum_{k=1}^{N} (u_{ik}^{*m} \mathbf{K}(\mathbf{z}_{k}, \mathbf{v}_{i}) \mathbf{z}_{k})}{\sum_{k=1}^{N} u_{ik}^{*m} \mathbf{K}(\mathbf{z}_{k}, \mathbf{v}_{i})}.$$
 (17)

KPyFCM algorithm is displayed in Algorithm 1.

2.5 Proposed Methodology

Figure 1 displays the proposed scheme in this paper for condition monitoring, which has two stages. The first stage is developed offline and the second one online.

In the off-line stage, the fuzzy classifier is trained by using a training database with historical data representative of the *l* operation modes or classes of the process (normal operation and faults).

Also, the mean vectors (\overline{Ob}_{C_l}) and the mode vectors (Ob_{MC_l}) of these classes are calculated.

The training database must be preprocessed before its use to eliminate the outliers which affect the satisfactory obtaining of the mean vectors.

In the online stage, each received observation is analyzed to determine if it has missing variables.

In negative case, the classification process of the observation is carried out.

In affirmative case, the imputation process is executed and then the observation is classified.





Fig. 7. Analysis of the ROC curve

Imputation process: If an observation with missing variables is received, the online imputation process is executed in the following form:

- An observation $Ob \in R^{1 \times p}$ (p: number of measured variables), arrives with rlost variables. The missing variables in the observation and in the mean vectors (\overline{Ob}_{C_l}) are excluded.

The Euclidean distances from the observation $\check{Ob} \in R^{1 \times p-r}$ to the mean vectors $\check{Ob}_{C_l} \in R^{1 \times p-r}(d_l = \check{Ob} - \check{Ob}_{C_l})$ are calculated.

The observation is assigned to the class that corresponds to the minimum distance $(\min(d_l))$. If considered, other types of distance measures can be used.

- The imputation process is executed.

Classification process: If the observation has no missing variables or when the missing variables have been estimated, it is assigned to a class

Computación y Sistemas, Vol. 27, No. 1, 2023, pp. 223–235 doi: 10.13053/CyS-27-1-4534 by the classification algorithm of the condition monitoring system.

If the degree of overlap among some classes is high, it is possible that the observation with the estimated variables will be classified in a different class than the one used for the imputation.

For the classification by using statistical classifiers, each observation is compared with the center of each class by using a measure of similitude to determine the class to which the observation belongs.

When fuzzy classification algorithms are used, the classification is made by using Eq. (18):

$$C_i = \{i : \max\{u_{ik}^*\}, \forall i, k\}.$$
 (18)

3 Study Case and Experimental Design

3.1 Case study: DAMADICS

To validate the proposed methodology, the DAMADICS (Development and Application of



Fig. 8. Faults classification (%) for the DAMADICS process

Methods for Actuator Diagnosis in Industrial Control Systems) test problem will be used.

It represents an intelligent electro-pneumatic actuator widely used in industries [4].

The information and the data sets related to this benchmark can be found in the URL¹. Table 1 shows the operation modes evaluated in the actuator and the measured variables used.

Selected faults occur in different parts of the actuator. In the case of faults F15 and F19, their patterns overlap, which makes difficult the satisfactory classification.

3.2 Design of Experiments

Table 2 shows the characteristics of the training database used, which is free of outliers, noise and missing variables.

K-cross-validation method was used in the training and validation. K = 5 was selected (800 observations for training and 200 for validation).

The database used for the experiments related to the online stage had 400 new observations of each operation mode which were not used in the training for a total of 2400 observations. Each experiment was repeated 100 times to ensure repeatability of results. The average of 100 results was considered as final result.

The data sets used in the experiments to evaluate the performance of the condition monitoring system had the following characteristics:

- Data set without missing information.
- Data set with 1 randomly missing variable per observation.
- Data set with a random number of missing variables between 0 and 2.

In addition, experiments for a) Observations without noise, b) Observations with 2 % of white noise and zero mean, and c) Observations with 5 % of white noise and zero mean were developed in each variant.

The values of the parameters used for the applied algorithms were: Number of iterations = 100, $\epsilon = 10^{-5}$, m = 2, $\sigma = 50$. This parameters were selected according to the experience in previous works [20].

¹http://diag.mchtr.pw.edu.pl/damadics/

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4 Results and Discussion

4.1 Online Recognition Stage

It is very important to analyze the quality and the performance of a condition monitoring system. A widely used tool for this purpose is the confusion matrix (CM), which permits the analysis of the performance of classification algorithms.

The values CM_{rs} for $r \neq s$ in the CM show the number of observations of the operation mode r that the classifier algorithm misclassifies in the operation mode s.

Then, all the necessary information to analyze the performance of a condition monitoring system can be obtained from the confusion matrix. In the paper, the metrics shown in the Eqs. (19), and (20) are used for studying the robustness of the proposed condition monitoring system:

Sensitivity =
$$\frac{\text{TP}}{\text{TP} + \text{FN}}$$
, (19)

Specificity
$$= \frac{TN}{FP + TN}$$
, (20)

where TP-true positive rate (%), FN - False Negative rate (%), TN- True Negative rate (%), and FP- False positive rate (%).

Sensitivity is the proportion of observations of positive class (NOC) that are classified as positive (TP) and the Specificity is the proportion of observations with the negative class (AOC), classified as negative (TN).

The Receiver Operating Characteristic (ROC) curve is the graphical way of representing these values at a variety of thresholds. It shows the relation Sensitivity **vs.** (1-Specificity), in a two-dimensional graph.

The area under a ROC curve is greater when the separability among the classes is satisfactory. If the confusion among classes increases, the area under a ROC curve (AUC) decreases.

Figures 2-6 show the classification results for the faults 1, 7, 12, 15 and 19 by using the KPyFCM algorithm for DAMADICS process. They show a global classification percentage obtained for each data set.

Algorithm	No.Wins	Ranking
KFCM	0	3
KIFCM	1	2
KPyFCM	2	1

On the other hand, from the values of TP , FN , TN , and FP shown in Table 3, it is possible to obtain the results of the Sensitivity and (1-Specificity) for observations with a random number of missing variables between 0 and 2 using imputation with mean values.

The values displayed in the Table 4 are used to build the ROC curves associated to the analyzed faults.

The combination of high values of the Sensitivity metric and low values of the (1-Specificity) metric demonstrate a successful performance. Fig. 7 displays an example of the ROC curves for faults 1, 7, 12, and 15.

The excellent results obtained demonstrate the high robustness and validity of the proposed condition monitoring scheme, even when the process is disturbed by noise in the measurements and missing data.

4.2 Comparison with Other Similar Algorithms

Figure 8 shows the classification results by using the KFCM [21], KIFCM [23] and KPyFCM algorithms, considering a random number of missing variables between 0 and 2 using imputation with mean values and 5% of noise.

To establish if there are significant differences among the results of various algorithms, it is necessary to use statistical tests [7, 14].

4.2.1 Statistical Tests

First, the Friedman statistical test was applied, and as a result, the null hypothesis was rejected. This result indicates that at least one algorithm achieves results that are significantly different from the rest. Subsequently, the Wilcoxon test for a level of significance $\alpha = 0.05$ was applied. The result is shown in Table 5 where it is evident that the KPyFCM algorithm is the winning algorithm.

5 Conclusions

This paper has presented a condition monitoring scheme for industrial plants characterized for its robustness versus noise and missing variables in the measurements.

In the experiments, different noise levels and distinct quantities of missing variables in the measurements obtained from the plant were combined. The high performance obtained in the classification shows the high robustness grade of the analyzed scheme. Another advantage of the proposal is to execute online the imputation process for each observation obtained from the plant that has missing variables.

This is a distinctive characteristic with respect to most proposals present in the scientific bibliography, which need to accumulate a group of observations before executing the imputation.

The implemented imputation methods are very simple so they have low computational complexity. This allows for their use in processes with short sampling times, which represents another advantage of the proposal.

For the classification process, the KPyFCM algorithm was implemented. In this algorithm, the capacity of the function kernel to achieve greater separability among the classes was combined with the possibility of the Pythagorean membership functions for using a larger set of numeric values for assigning the membership degree to an observation.

This provided an algorithm with high robustness that allowed the condition monitoring scheme to obtain high-performance levels versus noisy observations and with missing variables. The proposed condition monitoring scheme was validated using the DAMADICS process benchmark where it obtained a high performance.

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